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Exercise 1 (12mks)

Let's study the move in the interval $I = [0;\ +\infty[$ on which the horary is:

$\varphi(t) = cos\frac{\pi}{4}t + sin\frac{\pi}{4}t + 1$.

1)What is the nature of that movement?

2) Give its magnitude and horde.

3) Define its centre and deduce the reference $(0,\vec{u})$.

4) Calculate its period and its phase.

We change now the reference and let be $(C,\vec{u})$ the new reference defined so that $x_1(t) = A(cos\omega t + \varphi_0)$.

5) Determine $A, \omega\ and\ \omega_0$.

Let us study the position of the point M on the reference $(0,\vec{u})$ corresponding to M_0 in the reference$(C,\vec{u})$.

6) Calculate the position of $x_1(0)$, its algebraic motion $v(0)$ and its acceleration $\gamma(0)$.

7) Calculate the product, $v(0).\gamma(0)$. What then to conclude?

8) Deduce the direction of the movement at this initial time.

9) Determine the points in which the movement is will be equal to zero in the interval $[0; 8]$.

10) Calculate $x_1(1) and\ x_1(5)$.

11) Draw the diagram of that function within its period.

Exercise 2 (8mks)

A machine-tool manufactures cylinders. We measure the divergence in tenth of millimeter between the diameter of the obtained cylinders and the value of adjustment of the machine.

We suppose that the divergence follows an exponential law with parameter $\lambda = 1{,}5$

If the divergence is less than 1, the cylinder will be accepted. If the divergence is between 1 and 2, we proceed to a rectification which allows us to admit the cylinder in 80% of the cases. If the divergence is greater than 2, the cylinder will be refused.

1) We draw in random on cylinder in the production.

a) What is the probability, with a precision of 10^{-3}, so that the cylinder should be accepted?

b) Knowing that it has been accepted, what is the probability so that it should have been rectified?

2) We take in an independent way ten cylinders of the production. We suppose that the number of cylinders is sufficiently important to assimilate that draw to a sequential draw drawback.

a) What is the probability that the ten cylinders should be accepted?

b) What is the probability that at least one cylinder should be refused?

($Hints$.for a continuous random variable following an exponential law with parameter β, we have

$$P(X \leq a) = \int_0^a \lambda e^{-\lambda t}\, dt$$)

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EXERCISE I(7markes)

A) Let f be the function defined on $]0;+\infty[$ by $f(x) = x - 2 + \frac{1}{2}In(x)$

1) a) Calculate the limits of f in 0 and $+\infty$.

b) Calculate $f^{'}$ and give the variation sense of$f$.

2) a) Show that the equation $f(x) = 0$ has a unique solution denoted on $]0;+\infty[$.

Give the approximate value of α at 10^{-2} near.

b) Study the sign of $f(x)$.

B) Let g be the function defined $[0;+\infty[$ by $g(x) = -\frac{7}{8}x^2 + x - \frac{1}{4}x^2 In(x)$. $\forall x > 0$ and $g(0) = 0$.

1) a) Study the continuity and differentiability of g in 0.

b) Determine the limit of g in $+\infty$.

2) Let's consider $g^{'}$ the derivative of the function g. $\forall x > 0$, calculate $g^{'}(x)$ and verify that $g^{'}(x) = xf\left(\frac{1}{x}\right)$

3) Deduce the sign of $g^{'}(x)$ Draw the variation table of g.

4) Give the equations of tangents to the curve denoted (C) of g at the points of x-coordinates 0 and 1.Plot (C) and the previous tangents.

EXERCISE II (2+2=4)

Evaluate

1) $\int_0^{\sqrt{3}} x^2 arctan(x)dx$

2) $\int_0^1 \frac{1}{(1+x^2)^2} dx$

***EXERCISE III* (3markes)**

Solve the following differential equation $y^{'} + y = 2\cos(x) + (x+1)e^{-x}$.

EXERCISE IV (6markes)

Let's consider the numerical sequence (U_n) and (V_n) defined by $U_0 = 2$ $\forall \in IN$ $V_n = \frac{2}{U_n}$ and $U_{n+1} = \frac{U_n + V_n}{2}$

1) Calculate V_0; $U_1; V_1; U_2; V_2$. Give the results in the form of non-reducible fraction.

2) Show that (U_n) and (V_n) are bounded above by 2 bounded below by 1.

3) Show that $\forall n \in IN$ $U_{n+1} - V_{n+1} = \frac{(U_n - V_n)^2}{2(U_n + V_n)}$.

4) Show that $\forall n \epsilon IN$ $U_n \geq V_n$.

5) Show that (U_n) is decreasing and (V_n) is increasing.

6) Show that $\forall n \in \mathbb{N}$, , $(u_n - v_n) \leq 1$. Deduced that $(u_n - v_n)^2 \leq (u_n - v_n)$.

7) a) Show that $\forall n \in \mathbb{N}, u_{n+1} - v_{n+1} \leq \frac{1}{4}(u_n - v_n)$.

b) Show that $\forall n \in \mathbb{N}, u_n - v_n \leq \frac{1}{4^n}$

8) Show that (u_n) and, (v_n), converges to the same limit.