JULY 2011

Series: COMPUTER SCIENCE

Exercise 1 $(\boldsymbol{12mks})$

Let's study the motion in the interval $I = [0; +\infty\ ]$ on which the horary is

$\varphi(t) = cos\frac{\pi}{4}t + sin\frac{\pi}{4}t + 1.$

1. Let's state the nature of motion.

$$The\ motion\ of\ \varphi(t)\ is\ sinusoidal.$$

2. Let's give its magnitude and horde

⋆ Magnitude. $\varphi(t) = cos\frac{\pi}{4}t + sin\frac{\pi}{4}t + 1$

$\varphi'(t) = -\frac{\pi}{4}sin\frac{\pi}{4}t + \frac{\pi}{4}cos\frac{\pi}{4}t$, when $\varphi'(t) = 0$ we have

$-\frac{\pi}{4}sin\frac{\pi}{4}t + \frac{\pi}{4}cos\frac{\pi}{4}t = 0 \ \ \Leftrightarrow sin\frac{\pi}{4}t = cos\frac{\pi}{4}t = cos\left(\frac{\pi}{4}t - \frac{\pi}{2}\right)$

$$\begin{cases} \frac{\pi}{4}t = \frac{\pi}{4}t - \frac{\pi}{2} + 2k\pi; k \in \mathbb{Z} \\ \frac{\pi}{4}t = -\frac{\pi}{4}t + \frac{\pi}{2} + 2k\pi;\ \ k \in \mathbb{Z} \end{cases} \Leftrightarrow \frac{\pi}{2}t = \frac{\pi}{2}t + 2k\pi \Leftrightarrow t = 1 + 4k;\ k \in \mathbb{Z}$$

When $k = 0, t = 1, \varphi(1) = cos\frac{\pi}{4} + sin\frac{\pi}{4} + 1 = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} + 1$

Therefore

$$magnitude\ of\ \varphi(t), \varphi(1) = \sqrt{2} + 1$$

⋆ Horde or angle Its horde or angle is $\frac{\pi}{4}rads^{-1}$

3. Let's define the centre and deduce the reference $(0;\ \vec{u})$

The centre is $C(0,1) and\ \varphi(1) - 1 = 1\ and\ (0; \vec{u})\ is\ the\ origin$

4) Lets calculate its period, T and its phase. $T = \frac{2\pi}{\omega} = \frac{2\pi}{\frac{\pi}{4}} = 8, T = period$

We have $\frac{\pi}{4}t + sin\frac{\pi}{4}t = \sqrt{2}\left(\frac{\sqrt{2}}{2}cos\frac{\pi}{4}t + \frac{\sqrt{2}}{2}sin\frac{\pi}{4}t\right) = \sqrt{2}cos\left(\frac{\pi}{4}t - \frac{\pi}{4}\right)$. Thus

$$Period\ is\ T = 8\ and\ it'sphase\ is -\frac{\pi}{4}$$

Let $(C, \vec{u})$ be a new reference defined by $x_t(t) = Acos(\omega t + \varphi_0)$

5) Let's determine $A, \omega\ and\ \varphi_0$Here $x_1(t) = A\cos(\omega t + \varphi_0) \equiv \sqrt{2}cos\left(\frac{\pi}{4}t - \frac{\pi}{4}\right)$

Thus

$$A = \sqrt{2}, \omega = \frac{\pi}{4}\ and\ \varphi_0 = -\frac{\pi}{4}$$

∗Let's study the position of the point M on the reference $(C;\ \vec{u})$corresponding to $M_0\ in\ the\ reference\ (C;\ \vec{u})$.

6. Calculate the position $x_1(0), its\ algebraic\ motion\ v(0)\ and\ its\ acceleration\ y(0)$

⋆ $x_1(0) = \sqrt{2}cos\left(\frac{\pi}{4}.0 - \frac{\pi}{4}\right) = \sqrt{2}cos\left(-\frac{\pi}{4}\right) = 1$...........(1)

⋆ $x'_1 = v(t) = \left[\sqrt{2}cos\left(\frac{\pi}{4}t - \frac{\pi}{4}\right)\right]' = -\sqrt{2}\frac{\pi}{4}sin\left(\frac{\pi}{4}t - \frac{\pi}{4}\right)$

$v(0) = -\sqrt{2}.\frac{\pi}{4}\sin\left(-\frac{\pi}{4}\right) = -\sqrt{2}.\frac{\pi}{4}\left(\frac{-\sqrt{2}}{2}\right) = \frac{\pi}{4}\text{rads}^{-1}$ ... ... (2)

⋆ $x''_1(t) = v'(t) = y(t) = \left[-\sqrt{2}.\frac{\pi}{4}sin\left(\frac{\pi}{4}t - \frac{\pi}{4}\right)\right]' = \frac{\pi^2}{16}.cos\left(\frac{\pi}{4}t - \frac{\pi}{4}\right)$

$y(0) = -\frac{\pi^2}{16}.\sqrt{2}\ cos\left(-\frac{\pi}{4}\right) = -\frac{\pi^2}{16}rads^{-2}$ ..... (3)

Thus

$$\boxed{x'_1(t) = 1, v(0) = \frac{\pi}{4}rads^{-1}and\ y(0) = -\frac{\pi^2}{16}}$$

7. Let's calculate the product $v(0).y(0)$

We've from equations (2) and (3) that $v(0).y(0) = \left(\frac{\pi}{4}\right)\left(\frac{-\pi^2}{16}\right) = -\frac{\pi^3}{64}$

Thus

$$\boxed{v(0).y(0) = -\frac{\pi^3}{64} < 0, hence\ the\ motion\ is\ retard\ at\ t = 0s}$$

8. Let's deduce the direction of the motion at initial time.

At initial time we have $x(0) = 1\ and\ v(0) = \frac{\pi}{4}rads^{-1}$ thus

The motion is in the positive direction

9. Let's determine the points on which the motion will be zero in $[0; 8]$.

We have $x'_1(t) = \sqrt{2}cos\left(\frac{\pi}{4}t - \frac{\pi}{4}\right);\ x_1(0) = 0 \Leftrightarrow cos\left(\frac{\pi}{4}t - \frac{\pi}{4}\right) = cos\frac{\pi}{2} \Leftrightarrow \left(\frac{\pi}{4}t - \frac{\pi}{4}\right) = 2k\pi \pm \frac{\pi}{2} \Leftrightarrow t = \frac{4}{\pi}\left(2k\pi \pm \frac{\pi}{2}\right) = 8k \pm 2.$

$$t = \begin{cases} 8k + 2 \\ 8k - 2 \end{cases}\quad k\epsilon\mathbb{Z}$$

The values of t are$t = 8k + 2$ *and* $t = 8k - 2, k\epsilon\mathbb{Z}$.

10) Let's calculate $x_1(1)$ and $x_1(5)$.

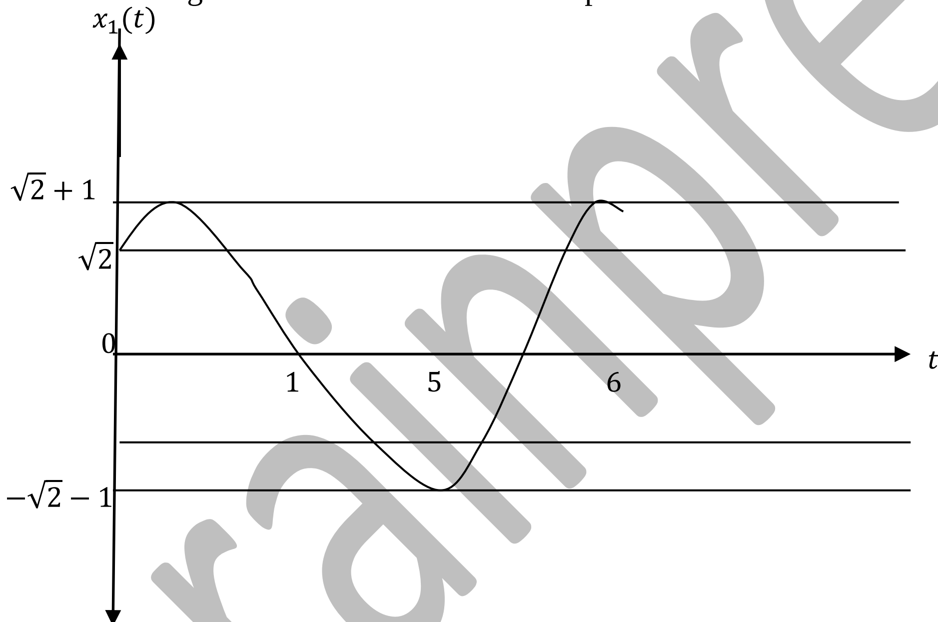
$x_1(t) = \sqrt{2}cos\left(\frac{\pi}{4}t - \frac{\pi}{4}\right) \Leftrightarrow x_1(1) = \sqrt{2}cos\left(\frac{\pi}{4} - \frac{\pi}{4}\right) = \sqrt{2}\cos(0)$

$x_1(1) = \sqrt{2}$…………….(a)

$x_1(5) = \sqrt{2}cos\left(\frac{5\pi}{4} - \frac{\pi}{4}\right) = \sqrt{2}cos(\pi) = -\sqrt{2}\cos(0) = -\sqrt{2}$. Thus

$\boxed{x_1(1) = \sqrt{2}\ and\ x_1(5) = -\sqrt{2}}$

11). let's draw the diagram of the function within its per



Exercise 2(8pts)

Let X be the variation. And that the following events are given

$A\ ''the$ cylinder is accepted$''$ and B ''the cylinder undergoes a correction''.

1) a. the cylinder is accepted if the variation is inferior to 1 or if one proceeds, to a correction when the variation lies between 1 and 2 and if it's accepted

$P(A) = P(X < 1) - P(A \cap B) = P(A) = P(X < 1) - P\left({}^{A}/_{B}\right)P(B)$

P(A< 1) $= \int_0^1 \lambda e^{-\lambda t}dt = (-e^{-1})_0^1 = 1 - e^1 = 0.776.$

P$(B \cap A) = P\left({}^{A}/_{B}\right)P(B) = 0.8xP(B)$

P(B)=P(1$\leq X \leq 2) = P(X \leq 2) - P(X \leq 1)$

$$P(X \leq 2) = \int_0^2 \lambda e^{-\lambda t}\, dt = \int_0^2 2e^{-2t}dt = 1 - e^{-0.8} = 0.950.$$

$P(X) = 0.950 - 0.776 = 0.174$

$P(A \cap B) = 0.139\ and\ P(A) = 0.915$. Thus,

$\boxed{P(A) = 0.915}$

$* P\left({}^{B}/_{A}\right) = \frac{P(B\cap A)}{P(A)} = \frac{0.139}{0.915} = 0.152.$ $\boxed{P\left({}^{B}/_{A}\right) = 0.152}$ Thus,

2)a) let's find the probability that ten cylinders are accepted.

We have $[P(A)]^{10} = [0.915]^{10} = 0.411.$ Thus

$\boxed{[P(A)]^{10} = 0.411}$

b) Let's find the probability that at least a cylinder is refused.

Let E be the event ' At least a cylinder is refused'. The contrary event is that " the ten cylinders are accepted''. Thus we have that

$P(E) = 1 - [P(A)]^{10} = 1 - 0.411 = 0.589$. Implying that

$\boxed{P(E) = 0.589}$