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EXERCISE I (5pts)

The complex plane is related the direct orthogonal frame $(0,\overrightarrow{u},\ \vec{v})$, graphic unit: 2cm.

1. Draw the circle whose centre is 0 and the radius 1 and
2. Indicate points $A, B\ and\ D$ with respective affixes $\sqrt{3}+i, \sqrt{3}-i\ and -\frac{1}{2}+\frac{\sqrt{3}}{2}i$ (1marke)
3. We consider the rotation R with centre 0 and angle $\frac{\pi}{3}$ and the translation T of vector with affix IR

a) Determine affixes $Z_{A'}$ and Z_B of points A' and B,respective image of A and B under the rotation R (0.75markes)

b) Determine the affix $Z_{D'}\ of\ D'$ where D' is image of D under the translation T (0.75markes)

c) Indicate points, $A', B'\ and\ D'$. Determine the argument of the complex number $\frac{Z_{A'}-Z_{B'}}{Z_{D'}}$

Prove that the line (OD') is a median of triangle$(OA'B')$.

EXERCISES 2 AND 3 EXIST IN 2010 AHEAD.

EXERCISE 4(4PTS)

This exercise has 4 affirmations. Indicate for each of them if it is true or false and justify your answer. Given the function defined by $f(x) = In\left(\frac{2x+1}{x-1}\right)$

1. f is defined on $]1; +\infty[$ (0.5markes)
2. $f'(x) = -\frac{1}{(x-1)^2} In\left(\frac{2x+1}{x-1}\right)$ (1markes)
3. $Line\ x = 1$ is asymptote ton the representative curve of f.
4. The representative curve of f admit a horizontal asymptote.(0.75mark)

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*Exercise I (4marks***)**

Given that the sequences U_n defined by U_0=2, U_1 =3 and U_n =$\frac{4U_{n-1} - U_{n-2}}{3}$ and V_n defined by V_n =U_n-V_n .n∈ N^*

e) Show that (V_n) is a geometrical sequence in which you will determine the first term and common ratio.

f) Calculate the general expression in terms of n.

g) Calculate S_n =V_1+V_2+ . . . +V_n in terms of n.

h) Show that that sequence (U_n) converges and specify its limit.

Exercise II (3marks)

1) By using the substitution u=1+cosh(x).Evaluate$\int_0^{\cosh^{-1}(2)} \frac{\tanh(x)}{1+\cosh(x)}\, dx$, Leaving your answer in terms of natural logarithms. 2) Show that $\frac{d}{dx}(\tanh^{-1}(x))$= $\frac{1}{1-x^2}$ .Hence, or otherwise show that $\int_0^{\frac{1}{2}} \tanh^{-1}(x)\, dx =$ $\frac{1}{4}$in$\left[\frac{27}{16}\right]$

Exercise III (2marks)

Determine the Cartesian equation of the plane:

c) Passing through P(-2;6;7) and has as normal vector n(0;3;0).

d) Passing through P(-6;10;16) and is perpendicular to the right-hand side of AB given A(1;0;5) and B(3;-3;2)

EXERCISE IV (4marks)

6) Find the square root of the complex number: Z_1=5+12i.

7) Find the modulus and argument of the complex number: Z_2=$\frac{(1+i)^2}{(-1+i)^4}$

8) Given that Z_3=1+i$\sqrt{3}$ represent the complex numbers $Z_3 Z^*{}_3$ and $\frac{Z_3}{Z^*{}_3}$ as vectors on an Argand diagram where $Z^*{}_3$ is the complex conjugate of Z_3 .

EXERCISE V (3marks)

Calculate the following quantities

a)$\int \frac{3x+5}{x^2+x+1}\, dx$ b)$\int \frac{3x+7}{\sqrt{-2x^2+x+1}}\, dx$

EXERCISE VI (4marks)

A continuous random

$$f(x) = \begin{cases} cx^3(1-x^2) \\ 0, elsewhere \end{cases}, if\ 0 \leq x \leq 1$$

Find

d) The value of the constant c

e) The mean and variance of X

f) The mode of X

Given that m is the median of X, show that $4m^6 - 6m^4 + 1 = 0$