2011

Exercise 1 (10mks)

a) Solve the system of equations $\begin{cases} log(x^2+y^2) = 1 + log8 \\ \log(x+y) - \log(x-y) = log3. \end{cases}$ where $= \frac{Inx}{In10}$.

b) Show that: $1 - \cos\left(\frac{3}{2}\pi - 3\alpha\right) - sin^2\frac{3}{2}\alpha + cos^2\frac{3}{2}\alpha = 2\sqrt{2}.cos\frac{3}{2}\alpha.\sin\left(\frac{3}{2}\alpha + \frac{\pi}{4}\right).$

c) Solve the inequation $(\log_2(x))^4 - \left(\log_{\frac{1}{2}}\left(\frac{x^3}{8}\right)\right)^2 + 9.\log_2\left(\frac{32}{x^2}\right) < 4.\left(\log_{\frac{1}{2}}(x)\right)^2,$

where $\log_a(b) = \frac{In(a)}{In(b)}$.

d) Evaluate $G = \frac{\left(\frac{a+a^{\frac{3}{4}}b^{\frac{1}{2}}+b^{\frac{3}{2}}a^{\frac{1}{4}}+b^2}{a^{\frac{1}{2}}+2a^{\frac{1}{4}}b^{\frac{1}{2}}+b}(\sqrt[4]{a}+\sqrt{b})+\frac{3\sqrt{b}\left(a^{\frac{1}{2}}-b\right)}{a^{-\frac{1}{4}}\left(a^{\frac{1}{4}}-\sqrt{b}\right)}\right)^{\frac{-1}{3}}}{(\sqrt[4]{a}+\sqrt{b})^{-1}}.$

Exercise 2 (10mks)

Consider, $P(z) = z^3 - (4+3i)z^2 + (1+9i)z + 2 - 6i.$

1) Show that there exist $z_0 \in IR\ and\ z_1 \in iIR\ which\ verify\ P(z_0) = P(z_1) = 0.$

2) Find the complex number z_2 such that $P(z) = (z-z_0)(z-z_1)(z-z_2).$

3) Let A, B, and C three points of the plane $z_0 = z_A,\ z_B = z_1, and\ z_C = z_2$.

a) Find the nature and the characteristical elements of the transformation s of the plane such that $S(A) = B\ and\ S(B) = C$.

b) Find $S(D)\ where\ D$ is the straight line defined by:$4x - 2y + 1 = 0.$

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Exercise 1 (12mks)

Let's study the move in the interval $I = [0;\ +\infty[$ on which the horary is:

$\varphi(t) = cos\frac{\pi}{4}t + sin\frac{\pi}{4}t + 1$.

1)What is the nature of that movement?

2) Give its magnitude and horde.

3) Define its centre and deduce the reference $(0, \vec{u})$.

4) Calculate its period and its phase.

We change now the reference and let be $(C, \vec{u})$ the new reference defined so that $x_1(t) = A(cos\omega t + \varphi_0)$.

5) Determine $A, \omega\ and\ \omega_0$.

Let us study the position of the point M on the reference $(0, \vec{u})$ corresponding to M_0 in the reference$(C, \vec{u})$.

6) Calculate the position of $x_1(0),$ its algebraic motion $v(0)$ and its acceleration $\gamma(0)$.

7) Calculate the product, $v(0).\gamma(0)$. What then to conclude?

8) Deduce the direction of the movement at this initial time.

9) Determine the points in which the movement is will be equal to zero in the interval $[0; 8]$.

10) Calculate $x_1(1) and\ x_1(5)$.

11) Draw the diagram of that function within its period.

Exercise 2 (8mks)

A machine-tool manufactures cylinders. We measure the divergence in tenth of millimeter between the diameter of the obtained cylinders and the value of adjustment of the machine.

We suppose that the divergence follows an exponential law with parameter $\lambda = 1,5$

If the divergence is less than 1, the cylinder will be accepted. If the divergence is between 1 and 2, we proceed to a rectification which allows us to admit the cylinder in 80% of the cases. If the divergence is greater than 2, the cylinder will be refused.

1) We draw in random on cylinder in the production.

a) What is the probability, with a precision of 10^{-3}, so that the cylinder should be accepted?

b) Knowing that it has been accepted, what is the probability so that it should have been rectified?

2) We take in an independent way ten cylinders of the production. We suppose that the number of cylinders is sufficiently important to assimilate that draw to a sequential draw drawback.

a) What is the probability that the ten cylinders should be accepted?

b) What is the probability that at least one cylinder should be refused?

($Hints$.for a continuous random variable following an exponential law with parameter β, we have

$P(X \leq a) = \int_0^a \lambda e^{-\lambda t}\, dt$)