***JULY 2009***

*Exercise I (4marks***)**

Given that the sequences $U_n$ defined by $U_0$=2, $U_1$ =3 and $U_n$ =$\frac{4U_{n-1}-U_{n-2}}{3}$ and $V_n$ defined by $V_n$ =$U_n$-$V_n$ .n$\in N^*$

a) Show that $(V_n)$ is a geometrical sequence in which you will determine the first term and common ratio.

b) Calculate the general expression in terms of n.

c) Calculate $S_n$ =$V_1$+$V_2$+ . . . +$V_n$ in terms of n.

d) Show that that sequence ( $U_n$) converges and specify its limit.

***Exercise II (3marks)***

1) By using the substitution u=1+cosh( $x$).Evaluate$\int_0^{\cosh^{-1}(2)} \frac{\tanh(x)}{1+\cosh(x)}\,dx$, Leaving your answer in terms of natural logarithms. 2) Show that $\frac{d}{dx}(\tanh^{-1}(x))$=$\frac{1}{1-x^2}$ .Hence, or otherwise show that $\int_0^{\frac{1}{2}}\tanh^{-1}(x)\,dx = \frac{1}{4}$in$\left[\frac{27}{16}\right]$

***Exercise III (2marks)***

Determine the Cartesian equations of the planes:

a) Passing through P(-2;6;7) and has as normal vector n(0;3;0).

b) Passing through P(-6;10;16) and is perpendicular to the right-hand side of AB given A(1;0;5) and B(3;-3;2)

***EXERCISE IV (4marks)***

1) Find the square root of the complex number: $Z_1$=5+12i.

2) Find the modulus and argument of the complex number: $Z_2$=$\frac{(1+i)^2}{(-1+i)^4}$

3) Given that $Z_3$=1+i$\sqrt{3}$ represent the complex numbers $Z_3Z^*{}_3$ and $\frac{Z_3}{Z^*{}_3}$ as vectors on an Argand diagram where $Z^*{}_3$ is the complex conjugate of $Z_3$ .

***EXERCISE V (3marks)***

Calculate the following quantities

a) $\int \frac{3x+5}{x^2+x+1}dx$ b) $\int \frac{3x+7}{\sqrt{-2x^2+x+1}}dx$

***EXERCISE VI (4marks)***

A continuous random

$f(x) = \begin{cases} cx^3(1-x^2) \\ 0, elsewhere \end{cases}, if\ 0 \leq x \leq 1$

Find

a) The value of the constant c
b) The mean and variance of X
c) The mode of X

Given that m is the median of X, show that $4m^6 - 6m^4 + 1 = 0$