OCTOBER 2012
MATHEMATICS F

Question 1

Given the complex numbers, $Z_1 = 3\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)$ and $Z_2 = 3\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)$.

1-Give the trigonometric form of the following complex numbers $Z_1;\ Z_2$ and $\frac{Z_1}{Z_2}$

2-Prove that for all integers n; Z^{12n} is a real number.

3-What are the real values of the $cos\frac{5\pi}{12}$ and $\sin\frac{5\pi}{12}$.

Question 2

Find the particular solution of the following differential equation$4f''(x) - 4f'(x) + f(x) = 0$ such that $f(0) = 4$ and the tangent to the curve of f at the point with $x = 2$ is horizontal.

Question 3

Solve for $(x, y)$the following system in the set of real numbers:

$$\begin{cases} x^2 + y^2 = 145 \\ \ln x + \ln y = \ln 72 \end{cases}$$

Question 4

Given the line $L: y = 2x + 4$ and the circle, $x^2 + y^2 - 4x - 6y - 12 = 0$.

1-The points of intersection of the line and the circle are;

a) $A(-2; 0); B(2; 8)$ b) $A(2; 0); B(-2; 8)$ c) $A(-2; 8)$ $B(0; 8)$ d) $A(0; -2); B(8; 2)$

2-The length of the cord cut off is

a) $4\sqrt{5}$ b) $5\sqrt{4}$ c) $\sqrt{68}$ d) 8

Question 5

Given the function, $f(x) = (2x^2 - 7x + 7)e^x$. Choose the correct answer.

1-$\lim_{x\to-\infty} f(x) = 0$; $\lim_{x\to+\infty} f(x) = +\infty$; $f\left(-\frac{1}{2}\right) = \frac{9}{\sqrt{e}}$

2-$\lim_{x\to-\infty} f(x) = -\infty$; $\lim_{x\to+\infty} f(x) = 0$; $f\left(-\frac{1}{2}\right) = \frac{9}{\sqrt{e}}$

3-The function f is positive and increasing in $\left]-\infty; -\frac{1}{2}\right]$

4-The function f is negative and increasing in $[2; +\infty[$

5-The minimum of the function f is 0

6-$f(x) \geq -e^2$; $\forall x \in \mathbb{R}$.

<u>**SESSION: 2012**</u>

EXERCIS E 1(1.5+1+1.5=4marks)

If $\int_0^{\pi/2} cos^n(t)\, dt, \forall n \geq 1$

a) Calculate I_0, I_1 and I_2

b) Show that $\forall\ n \geq 2$, $one\ has$ $nI_n = (n-1)I_{n-2}$,

c) Deduce the value of I_n $\forall n \geq 1$

EXERCISE 2(4marks)

Study the continuity of the following function,

$$f(x) = \begin{cases} x^2 , x \leq 0 \\ x,\ \ 0 < x \ \ < 2 \\ 4 - x \ ,x \geq 2 \end{cases}$$

2) Show that the function g is continuous and differentiable and its derivative is continuous

$$G(x) = \begin{cases} e^{1/x}, \ x < 0 \\ 0, if \ x = 0 \\ \cos(x) - 1, if x > 0 \end{cases}$$

EXERCISE 3(marks)

One gives $\mathrm{E}(X) = 2, find$

a) The value of p and q,

b) Vary(X),

c) The average and the variance of Y=5X+7.

X=x	0	1	2	3	4
P(X=x)	0.1	p	0.25	q	0.05

EXERCISE 4(9marks)

Part 1 (6marks)

Let f be the function defined on R by $\mathrm{f}(x) = \frac{3(x-1)^3}{3x^2+1}$ and let C its curve representative in the plane brought back to an orthogonal reference mark of unit 1cm.

1)Show that there exist a single triplet (amebic), that one will determine such as for any real

$$f(x) = ax + b + \frac{CX}{3x^2+1},$$

2) $Determin\ the\ limits\ of\ f\ at + \infty\ and$ $-\infty$. 3)$Show\ that\ f\ is\ differentiable\ and\ calculate\ its\ derivative.$ 4) $Draw\ the\ table\ of\ variation\ of\ f$ 5) Show that that the curve (C) has as asymptote oblique the line (D) of equation y= x −3

6) Study the relative positions of (C) and (D)

7) Give the equation of tangent (T) to (C) at the point of x-coordinate 0. Trace (D),(T) and (C)

8) Show that the curve has a centre of symmetry

9) Show that the equation $\mathrm{f}(x) = 1$ has single solution in IR one note α this solution.

10) Give the approximate value of α to 10^{-2} nearby excess.

Part 2 (3marks)

One considers the function g defined on IR by: $\mathrm{g}(x) = \frac{3(\sin(x)-1)^3}{3\sin^2 x+1}$

1) Show that g is differentiable on IR and calculate $g\prime(x)$

2) Draw the table of variation of g on $[-\pi; \pi]$

3) Plot on a new drawing the curve representative of g.

AUGUST 2011 CS(maroua)

Exercise 1 (10mks)

a) Solve the system of equations $\begin{cases} log(x^2 + y^2) = 1 + log8 \\ \log(x+y) - \log(x-y) = log3. \end{cases}$ where $= \frac{Inx}{In10}$.

b) Show that: $1 - \cos\left(\frac{3}{2}\pi - 3\alpha\right) - sin^2\frac{3}{2}\alpha + cos^2\frac{3}{2}\alpha = 2\sqrt{2}.cos\frac{3}{2}\alpha.\sin\left(\frac{3}{2}\alpha + \frac{\pi}{4}\right)$.

c) Solve the inequation $(\log_2(x))^4 - \left(\log_{\frac{1}{2}}\left(\frac{x^3}{8}\right)\right)^2 + 9.\log_2\left(\frac{32}{x^2}\right) < 4.\left(\log_{\frac{1}{2}}(x)\right)^2$,

where $\log_a(b) = \frac{In(a)}{In(b)}$.

d) Evaluate $G = \dfrac{\left(\frac{a+a^{\frac{3}{4}}b^{\frac{1}{2}}+b^{\frac{3}{2}}a^{\frac{1}{4}}+b^{2}}{a^{\frac{1}{2}}+2a^{\frac{1}{4}}b^{\frac{1}{2}}+b}\left(\sqrt[4]{a}+\sqrt{b}\right)+\frac{3\sqrt{b}\left(a^{\frac{1}{2}}-b\right)}{a^{-\frac{1}{4}}\left(a^{\frac{1}{4}}-\sqrt{b}\right)}\right)^{\frac{-1}{3}}}{\left(\sqrt[4]{a}+\sqrt{b}\right)^{-1}}$.

Exercise 2 (10mks)

Consider, $P(z) = z^3 - (4+3i)z^2 + (1+9i)z + 2 - 6i$.

1) Show that there exist $z_0 \epsilon IR$ and $z_1 \epsilon iIR$ $which$ $verify$ $P(z_0) = P(z_1) = 0$.

2) Find the complex number z_2 such that $P(z) = (z - z_0)(z - z_1)(z - z_2)$.

3) Let A, B, and C three points of the plane $z_0 = z_A$, $z_B = z_1$, and $z_C = z_2$.

a) Find the nature and the characteristical elements of the transformation s of the plane such that $S(A) = B$ and $S(B) = C$.

b) Find $S(D)$ $where$ D is the straight line defined by: $4x - 2y + 1 = 0$.

JULY: 2011

Exercise 1 (12mks)

Let's study the move in the interval $I = [0;\ +\infty[$ on which the horary is:

$\varphi(t) = cos\frac{\pi}{4}t + sin\frac{\pi}{4}t + 1$.

1)What is the nature of that movement?

2) Give its magnitude and horde.

3) Define its centre and deduce the reference $(0, \vec{u})$.

4) Calculate its period and its phase.

We change now the reference and let be $(C, \vec{u})$ the new reference defined so that $x_1(t) = A(cos\omega t + \varphi_0)$.

5) Determine A, ω and ω_0.

Let us study the position of the point M on the reference $(0, \vec{u})$ corresponding to M_0 in the reference$(C, \vec{u})$.

6) Calculate the position of $x_1(0)$, its algebraic motion $v(0)$ and its acceleration $\gamma(0)$.

7) Calculate the product, $v(0).\gamma(0)$. What then to conclude?

8) Deduce the direction of the movement at this initial time.

9) Determine the points in which the movement is will be equal to zero in the interval $[0; 8]$.

10) Calculate $x_1(1)$ and $x_1(5)$.

11) Draw the diagram of that function within its period.

Exercise 2 (8mks)

A machine-tool manufactures cylinders. We measure the divergence in tenth of millimeter between the diameter of the obtained cylinders and the value of adjustment of the machine.

We suppose that the divergence follows an exponential law with parameter $\lambda = 1{,}5$

If the divergence is less than 1, the cylinder will be accepted. If the divergence is between 1 and 2, we proceed to a rectification which allows us to admit the cylinder in 80% of the cases. If the divergence is greater than 2, the cylinder will be refused.

1) We draw in random on cylinder in the production.

a) What is the probability, with a precision of 10^{-3}, so that the cylinder should be accepted?

b) Knowing that it has been accepted, what is the probability so that it should have been rectified?

2) We take in an independent way ten cylinders of the production. We suppose that the number of cylinders is sufficiently important to assimilate that draw to a sequential draw drawback.

a) What is the probability that the ten cylinders should be accepted?

b) What is the probability that at least one cylinder should be refused?

($Hints$.for a continuous random variable following an exponential law with parameter β, we have

$$P(X \leq a) = \int_0^a \lambda e^{-\lambda t}\, dt$$)