JULY 2011

EXERCISE I(7markes)

A) Let f be the function defined on $]0;+\infty[$ by $f(x) = x - 2 + \frac{1}{2}In(x)$

1) a) Calculate the limits of f in 0 and $+\infty$.
   b) Calculate f' and give the variation sense of$f$.

2) a) Show that the equation $f(x) = 0$ has a unique solution denoted on $]0;+\infty[$.
   Give the approximate value of α at 10^{-2} near.
   b) Study the sign of $f(x)$.

B) Let g be the function defined $[0;+\infty[$ by $g(x) = -\frac{7}{8}x^2 + x - \frac{1}{4}x^2 In(x)$. $\forall x > 0$ and $g(0) = 0$.

1) a) Study the continuity and differentiability of g in 0.
   b) Determine the limit of g in $+\infty$.

2) Let's consider g' the derivative of the function g. $\forall x > 0,$ calculate $g'(x)$ and verify that $g'(x) = xf\left(\frac{1}{x}\right)$

3) Deduce the sign of $g'(x)$ Draw the variation table of g.

4) Give the equations of tangents to the curve denoted (C) of g at the points of x-coordinates 0 and 1.Plot (C) and the previous tangents.

EXERCISE II (2+2=4)

Evaluate

1) $\int_0^{\sqrt{3}} x^2 arctan(x)dx$
2) $\int_0^1 \frac{1}{(1+x^2)^2}dx$

EXERCISE III **(3*markes*)**

Solve the following differential equation $y' + y = 2\cos(x) + (x+1)e^{-x}$.

EXERCISE IV (6markes)

Let's consider the numerical sequence (U_n) and (V_n) defined by $U_0 = 2$ $\forall \in IN$ $V_n = \frac{2}{U_n}$ and $U_{n+1} = \frac{U_n + V_n}{2}$

1) Calculate V_0; $U_1; V_1; U_2; V_2$. Give the results in the form of non-reducible fraction.
2) Show that (U_n) and (V_n) are bounded above by 2 bounded below by 1.
3) Show that $\forall n \in IN$ $U_{n+1} - V_{n+1} = \frac{(U_n - V_n)^2}{2(U_n + V_n)}$.
4) Show that $\forall n \epsilon IN$ $U_n \geq V_n$.
5) Show that (U_n) is decreasing and (V_n) is increasing.
6) Show that $\forall n \in \mathbb{N}, , (u_n - v_n) \leq 1.$ Deduced that $(u_n - v_n)^2 \leq (u_n - v_n)$.
7) a) Show that $\forall n \in \mathbb{N}, u_{n+1} - v_{n+1} \leq \frac{1}{4}(u_n - v_n)$.
b) Show that$\forall n \in \mathbb{N}, u_n - v_n \leq \frac{1}{4^n}$
8) Show that (u_n) and, (v_n), converges to the same limit.