

JULY 2010

Exercise I (4mks)Given that $f(t) = 2^{-t}$,1-a Put $f(t)$ in the form $f(t) = e^{kt}$.1-b Evaluate $\int f(t)dt$ 2 The sequence $(U_n)_{n \geq 1}$ is defined by $U_n = \int_{-n}^n 2^{-t} dt$, show that (U_n) is geometrical sequence with ratio $\frac{1}{2}$.3 Given that: $S_n = \sum_{i=1}^n U_i$. Calculate S_n in terms of n .**Exercise II (3mks)**1) Solve the differential equation $\frac{d^2y}{dx^2} - 16y = 0$. Given that $y = 1, \frac{dy}{dx} = 0$, when $x = 0$.2) When a body falls under gravity in a medium in which the resistance is proportional to the speed, the relation between the speed V and the time t is given by: $\frac{dy}{dx} + kV = g$. Where g and k are constants. Prove that $V = \frac{g}{k} + Ae^{-kt}$, where A is a constant.3) Given that $v = 0$ when $t = 0$. find t when $v = \frac{g}{2k}$.**Exercise III (8mks)**

1) Calculate the following quantities:

$$a) \int \frac{2x}{x^4+1} dx \quad b) \int \left(\frac{1}{\sin^2(x)} + \frac{1}{\cos^2(x)} \right) dx \quad c) \int_0^1 \frac{e^x-1}{e^x+1} dx \quad d) \int_0^1 \frac{t}{1+t^4} dt.$$

Exercise IV (3mks)A transformation T of three dimensional space is defined by:

$$r' = (x', y', z'), r = (x, y, z), M \begin{pmatrix} 7 & 5 & 6 \\ 4 & 3 & 3 \\ 10 & 7 & k \end{pmatrix}, \text{ where } k \text{ is a constant.}$$

a) Find the value of k for which there is no inverse transformation.b) If $k = 9$, show that all the points (x, y, z) are transformed into points of the plane $2x' - y' - z' = 0$ c) If $k = 8$, find M^{-1} and hence, or otherwise, find point which is mapped into $(6, 2, 9)$.**Exercise V (4mks)**1) A discrete random variable X has the following probability distribution.

$(X = x)$	0	1	2	3	4
$P(X = x)$	0.12	p	0.4	q	0.08

Given that $E(X) = 2.02$, find:a) The value of p and q . b) $Var(X)$. c) The mean and variance of $Y = 3X - 2$ 2) The probability that an item produced by a certain machine is defective is 0.05. A quality control scheme consists of selecting 10 items from a large batch produced by the machine and the whole batch if three or more items are defective. Find the probability that a batch is rejected giving your answer correct to four decimal places, 10^{-4} .**Exercise VI (2mks)**Find the algebraic form of the complex number $= \frac{(1+i)^4}{(\sqrt{3}-i)^3}$.