OCTOBER 2012
MATHEMATICS F

Question 1

Given the complex numbers, $Z_1 = 3\left(-\frac{1}{2} + \frac{i\sqrt{3}}{2}\right)$ *and* $Z_2 = 3\left(\frac{\sqrt{2}}{2} + \frac{i\sqrt{2}}{2}\right)$.

1-Give the trigonometric form of the following complex numbers $Z_1;\ Z_2\ and\ \frac{Z_1}{Z_2}$

2-Prove that for all integers $n;\ Z^{12n}$ is a real number.

3-What are the real values of the $cos\frac{5\pi}{12}\ and\ \sin\frac{5\pi}{12}$.

Question 2

Find the particular solution of the following differential equation$4f''(x) - 4f'(x) + f(x) = 0$ such that $f(0) = 4$ and the tangent to the curve of f at the point with $x = 2$ is horizontal.

Question 3

Solve for $(x, y)$the following system in the set of real numbers:

$$\begin{cases} x^2 + y^2 = 145 \\ \ln x + \ln y = \ln 72 \end{cases}$$

Question 4

Given the line $L: y = 2x + 4$ and the circle, $x^2 + y^2 - 4x - 6y - 12 = 0$.

1-The points of intersection of the line and the circle are;

a) $A(-2; 0); B(2; 8)$ b) $A(2; 0); B(-2; 8)$ c) $A(-2; 8)$ $B(0; 8)$ d) $A(0; -2); B(8; 2)$

2-The length of the cord cut off is

a) $4\sqrt{5}$ b) $5\sqrt{4}$ c) $\sqrt{68}$ d) 8

Question 5

Given the function, $f(x) = (2x^2 - 7x + 7)e^x$. Choose the correct answer.

1-$\lim_{x\to-\infty} f(x) = 0$; $\lim_{x\to+\infty} f(x) = +\infty$; $f\left(-\frac{1}{2}\right) = \frac{9}{\sqrt{e}}$

2-$\lim_{x\to-\infty} f(x) = -\infty$; $\lim_{x\to+\infty} f(x) = 0$; $f\left(-\frac{1}{2}\right) = \frac{9}{\sqrt{e}}$

3-The function f is positive and increasing in $\left]-\infty; -\frac{1}{2}\right]$

4-The function f is negative and increasing in $[2; +\infty[$

5-The minimum of the function f is 0

6-$f(x) \geq -e^2;\ \forall x \in \mathbb{R}$.

SESSION: 2012

EXERCIS E 1(1.5+1+1.5=4marks)

If $\int_0^{\pi/2} cos^n(t)\, dt, \forall n \geq 1$

a) Calculate $I_0\,, I_1$ *and* I_2

b) Show that $\forall\, n \geq 2\,,$ *one has* $nI_n = (n-1)I_{n-2}\,,$

c) Deduce the value of I_n $\forall n \geq 1$

EXERCISE 2(4marks)

Study the continuity of the following function,

$$f(x) = \begin{cases} x^2\,, x \leq 0 \\ x,\ \ 0 < x \quad < 2 \\ 4 - x\ \ , x \geq 2 \end{cases}$$

2) Show that the function g is continuous and differentiable and its derivative is continuous

$$\mathrm{G}(x) = \begin{cases} e^{1/x},\ \ x < 0 \\ 0\,, if\ x = 0 \\ \cos(x) - 1, if x > 0 \end{cases}$$

EXERCISE 3(marks)

One gives $\mathrm{E}(X) = 2, find$

a) The value of p and q,

b) Vary(X),

c) The average and the variance of Y=5X+7.

X=x	0	1	2	3	4
P(X=x)	0.1	p	0.25	q	0.05

EXERCISE 4(9marks)

Part 1 (6marks)

Let f be the function defined on R by $\mathrm{f}(x) = \frac{3(x-1)^3}{3x^2+1}$ and let C its curve representative in the plane brought back to an orthogonal reference mark of unit 1cm.

1)Show that there exist a single triplet (amebic), that one will determine such as for any real

$$f(x) = ax + b + \frac{CX}{3x^2+1}\ ,$$

2) $Determin\ the\ limits\ of\ f\ at + \infty\ and$ $-\infty$. 3)$Show\ that\ f\ is\ differentiable\ and\ calculate\ its\ derivative.$ 4) $Draw\ the\ table\ of\ variation\ of\ f$ 5) Show that that the curve (C) has as asymptote oblique the line (D) of equation y= x −3

6) Study the relative positions of (C) and (D)

7) Give the equation of tangent (T) to (C) at the point of x-coordinate 0. Trace (D),(T) and (C)

8) Show that the curve has a centre of symmetry

9) Show that the equation $\mathrm{f}(x) = 1$ has single solution in IR one note α this solution.

10) Give the approximate value of α to 10^{-2} nearby excess.

Part 2 (3marks)

One considers the function g defined on IR by: $g(x) = \frac{3(\sin(x)-1)^3}{3\sin^2 x+1}$

1) Show that g is differentiable on IR and calculate $g'(x)$

2) Draw the table of variation of g on $[-\pi;\pi]$

3) Plot on a new drawing the curve representative of g.

AUGUST 2011 CS(maroua)

Exercise 1 (10mks)

a) Solve the system of equations $\begin{cases} log(x^2+y^2) = 1 + log8 \\ \log(x+y) - \log(x-y) = log3. \end{cases}$ where $= \frac{Inx}{In10}$.

b) Show that: $1 - \cos\left(\frac{3}{2}\pi - 3\alpha\right) - sin^2\frac{3}{2}\alpha + cos^2\frac{3}{2}\alpha = 2\sqrt{2}.cos\frac{3}{2}\alpha.\sin\left(\frac{3}{2}\alpha + \frac{\pi}{4}\right)$.

c) Solve the inequation $(\log_2(x))^4 - \left(\log_{\frac{1}{2}}\left(\frac{x^3}{8}\right)\right)^2 + 9.\log_2\left(\frac{32}{x^2}\right) < 4.\left(\log_{\frac{1}{2}}(x)\right)^2$,

where $\log_a(b) = \frac{In(a)}{In(b)}$.

d) Evaluate $G = \frac{\left(\frac{a+a^{\frac{3}{4}}b^{\frac{1}{2}}+b^{\frac{3}{2}}a^{\frac{1}{4}}+b^2}{a^{\frac{1}{2}}+2a^{\frac{1}{4}}b^{\frac{1}{2}}+b}(\sqrt[4]{a}+\sqrt{b})+\frac{3\sqrt{b}\left(a^{\frac{1}{2}}-b\right)}{a^{-\frac{1}{4}}\left(a^{\frac{1}{4}}-\sqrt{b}\right)}\right)^{\frac{-1}{3}}}{(\sqrt[4]{a}+\sqrt{b})^{-1}}$.

Exercise 2 (10mks)

Consider, $P(z) = z^3 - (4+3i)z^2 + (1+9i)z + 2 - 6i$.

1) Show that there exist $z_0 \epsilon IR$ and $z_1 \epsilon iIR$ $which$ $verify$ $P(z_0) = P(z_1) = 0$.

2) Find the complex number z_2 such that $P(z) = (z-z_0)(z-z_1)(z-z_2)$.

3) Let A, B, and C three points of the plane $z_0 = z_A$, $z_B = z_1$, and $z_C = z_2$.

a) Find the nature and the characteristical elements of the transformation s of the plane such that $S(A) = B$ and $S(B) = C$.

b) Find $S(D)$ $where$ D is the straight line defined by: $4x - 2y + 1 = 0$.