JULY 2010

Exercise I (4mk)

Given that $(t) = 2^{-t}$,

1-a Put $f(t)$ in the form $f(t) = e^{kt}$.

1-b Evaluate $\int f(t)dt$

2 The sequence $(U_n)_{n\geq 1}$ is defined by $U_n = \int_{-n}^{n} 2^{-t}dt$, show that (U_n) is geometrical sequence with ratio $\frac{1}{2}$.

3 Given that: $S_n = \sum_{i=1}^{n} U_i$. Calculate S_n in terms of n.

Exercise II (3mks)

1) Solve the differential equation $\frac{d^2y}{dx^2} - 16y = 0$. Given that $y = 1, \frac{dy}{dx} = 0, when\ x = 0$.

2) When a body falls under gravity in a medium in which the resistance is proportional to the speed, the relation between the speed V and the time t is given by: $\frac{dy}{dx} + kV = g$. Where $g\ and\ k$ are constants. Prove that $V = \frac{g}{k} + Ae^{-kt}$, where A is a constant.

3) Given that $= 0\ when\ t = 0. find\ t\ when\ v = \frac{g}{2k}$.

Exercise III (8mks)

1) Calculate the following quantities:

a) $\int \frac{2x}{x^4+1}dx$ b) $\int \left(\frac{1}{sin^2(x)} + \frac{1}{cos^2(x)}\right)dx$ c) $\int_0^1 \frac{e^x-1}{e^x+1}dx$ d) $\int_0^1 \frac{t}{1+t^4}dt$.

Exercise IV (3mks)

A transformation T of three dimensional space is defined by:

$r' = (x', y', z'), r = (x, y, z),\ M\begin{pmatrix} 7 & 5 & 6 \\ 4 & 3 & 3 \\ 10 & 7 & k \end{pmatrix}$, where k is a constant.

a) Find the value of k for which there is no inverse transformation.

b) If $k = 9$, show that all the points (x, y, z) are transformed into points of the plane$2x' - y' - z' = 0$

c) If $k = 8, find\ M^{-1}$ and hence, or otherwise, find point which is mapped into $(6, 2, 9)$.

Exercise V (4mks)

1) A discrete random variable X has the following probability distribution.

$(X = x)$	0	1	2	3	4
$P(X = x)$	0.12	p	0.4	q	0.08

Given that $E(X) = 2.02, find$:

a) The value of p and q. b)$Var(X)$. c) The mean and variance of $Y = 3X - 2$

2) The probability that an item produced by a certain machine is defective is 0.05. A quality control scheme consists of selecting 10items from a large batch produced by the machine and the whole batch if three or more items are defective. Find the probability that a batch is rejected giving your answer correct to four decimal places, 10^{-4}.

Exercise VI (2mks)

Find the algebraic form of the complex number $= \frac{(1+i)^4}{(\sqrt{3}-i)^3}$.

JULY 2009

EXERCISE I (5pts)

The complex plane is related the direct orthogonal frame $(0,\overrightarrow{u},\ \vec{v})$, graphic unit: 2cm.

1. Draw the circle whose centre is 0 and the radius 1 and
2. Indicate points $A, B\ and\ D$ with respective affixes $\sqrt{3}+i, \sqrt{3}-i\ and -\frac{1}{2}+\frac{\sqrt{3}}{2}i$ (1marke)
3. We consider the rotation R with centre 0 and angle $\frac{\pi}{3}$ and the translation T of vector with affix IR

a) Determine affixes $Z_{A'}$ and Z_B of points A' and B,respective image of A and B under the rotation R (0.75markes)

b) Determine the affix $Z_{D'}\ of\ D'$ where D' is image of D under the translation T (0.75markes)

c) Indicate points $A', B'\ and\ D'$. Determine the argument of the complex number $\frac{Z_{A'}-Z_{B'}}{Z_{D'}}$

Prove that the line (OD') is a median of triangle$(OA'B')$.

EXERCISES 2 AND 3 EXIST IN 2010 AHEAD.

EXERCISE 4(4PTS)

This exercise has 4 affirmations. Indicate for each of them if it is true or false and justify your answer. Given the function defined by $f(x) = In\left(\frac{2x+1}{x-1}\right)$

1. f is defined on $]1; +\infty[$ (0.5markes)
2. $f'(x) = -\frac{1}{(x-1)^2} In\left(\frac{2x+1}{x-1}\right)$ (1markes)
3. $Line\ x = 1$ is asymptote ton the representative curve of f.

The representative curve of f admit a horizontal asymptote.(0.75mark