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**UNIVERSITY OF BAMENDA**

**ECOLE NORMALE SUPERIEUR ANNEXE DE BAMBILI**

**COMMON ENTRANCE EXAMINATION JULY 2012**

**SECOND CYCLE TECHNICAL EDUCATION 3HOURS**

### EXERCISE 1

Determine whether the following sums converge or not. Justify your answers

1.  $\sum_{n=1}^{\infty} \frac{1}{2^n + 1}$

2.  $\sum_{n=1}^{\infty} \frac{2n+1}{n^3 + 1}$

3.  $\sum_{n=2}^{\infty} \frac{1}{\ln(n)}$

4.  $\sum_{n=1}^{\infty} \frac{99^n}{n!}$

5.  $\sum_{n=1}^{\infty} \frac{n}{2^n}$

### EXERCISE 2

Suppose that  $f$  is a differentiable function on the real line. In terms of the derivative  $f'$  of  $f$ , express the derivative of the following.

1.  $f(3x)$

2.  $f(x^2)$

3.  $f(\pi f(x))$

$[f(3-2f(x))]^4$

### EXERCISE 3

1. Verify that the differential equation:

$(2x + \sin y - ye^{-x})dx + (x \cos y + \cos y + e^{-x})dy = 0$  Is exact and find its solution curve

2. Solve the following:  $\frac{dy}{dx} + xy = x^2$

**EXERCISE 4**

Evaluate  $\iint_T xy dA$  over the triangle T with vertices (0, 0); (1, 0); and (1, 1)

**EXERCISE 5**

1. Find  $\int_0^3 f(x) dx$  where  $f(x) = \begin{cases} \sqrt{1-x^2} & \text{if } 0 \leq x \leq 1 \\ 2 & \text{if } 1 \leq x \leq 2 \\ x-2 & \text{if } 2 \leq x \leq 3 \end{cases}$
2. Find the total area A lying between the curve  $y = \sin x$  and  $y = \cos x$  from  $x=0$  to  $x=2\pi$

**UNIVERSITY OF BAMBILI H.T.T.T.C**

**CAMPUS: BAMBILI**

**ENTRANCE EXAMINATION,**

**JULY 2012**

**CYCLE: SECOND CYCLE**

**TECHNICAL TEACHING**

**SERIES: DATA PROCESSING**

**DURATION: 3H**

**COEFFICIENT: 4**

**TOPIC: APPLIED MATHEMATICS**

**Exercise I** (12 marks)

A company manufactures parts which it conditions per hundreds. Day labourer manufacture varies between 100 parts and 600 pieces. It is supposed that the

benefit, expressed in thousands of francs, concerning the quantity  $Q$  of parts manufactured, is given by:

$$f(q) = -2q^2 + 20q - 18 - 16 \ln q, \text{ With } q \text{ expressed in hundreds: } 1 \leq q \leq 6,5.$$

Part A. Economic study (2=1+0.5+0.5)

1. To calculate, in francs, the benefit carried out for a day labourer production of 100 coins then of 500 coins.
2. Calculate, in francs, the benefit corresponding to the manufacturing of the 201<sup>e</sup> parts.
3. One manufacturing 400 parts. Calculation, in francs, the average benefit carried out by part manufactured.

Part B. Theoretical study

Let  $f$  the function defined for all  $x \in [1, 6,5]$  by:

$$f(x) = -2x^2 + 20x - 18 - 16 \ln x.$$

1. General study (2,5=0,5+1+0,5+0,5 marks)

Calculation the derivative  $f'$  of  $f$ .

Show that the equation  $f'(x)=0$  has a solutions  $x_1$  and  $x_2$  which one will determine (with  $x_1 < x_2$ ).

Show that  $f'(x)$  has the same signs that  $-4(x-1)(x-4)$ .

Draw the table of variation of  $f$  on  $[1; 6,5]$ .

2. Particular study (2=1+ marks)

2.1. Show that the equation  $f(x) = 0$  has a solution  $\alpha$ , distinct from 1 and only one, in the interval  $[1; 6; 5]$ .

2.2. Prove that  $6,19 \leq \alpha \leq 6,20$ .

3. Graphic study. (2=1+1 marks)

Recopy and supplement the following table. The values of  $f(x)$  will be round with  $10^{-1}$  near.

$x$	1	2	3	4	5	6	6,5
$f(x)$							

Draw in the orthogonal frame, the curve of the function  $f$  on  $[1; 6; 5]$  Graphic

units 2cm on the X-axis. 1cm on the y-axis

4. The function  $g$  and  $G$  are respectively defined on  $]0, +\infty[$  by

$$g(x) = \ln(x) \quad \text{And} \quad G(x) = x \ln(x) - x$$

Check that  $G$  is a primitive of  $g$  on  $]0, +\infty[$  (0, 5 mark)

Deduce a primitive  $F$  of  $f$  on  $[1; 6; 5]$ . (0, 5 mark)

Part C. Return to the economic survey. (2,5=0,5+0,5+0,5+1)

1. Determine the units, maximum quantity parts to be manufactured so that it has benefit.

2. Determine, in unites, the ~~www.touslesconcours.info~~ manufacture in order to obtain the maximum benefit. To calculate in francs the benefit.
3. Reveal on the graph the preceding results.
4. The average benefit  $B_m$  carried out per hundreds of parts is expressed, in thousands of francs; by  $B_m = \frac{1}{5,5} \int_1^{6,5} f(x) dx$  calculate  $B_m$  with a margin of 100 francs.

### **EXERCISE II** (8 marks)

A company manufactures margarine breads industrially. Their packing carries in particular the following indication: "weight net: 500 grams"

#### 1) Theory of probability

It is estimated that the weight, in grams, of produced bread, is a random variable  $X$  which follows the normal law of average 520 and standard deviation 20

It is supposed that the weights of different the bread are independent from to each other

With packing, a bread is refused if its weight is lower or equal to 490 grams.

- a) a bread arrives at packing. Show that the decimal approximation round with  $10^{-4}$  close to the probability that it is refused is equal to 0.0668
- b) Two breads arrive at packing. Calculate the probability of each of the two following events.

A: "the two breads are refused"

B: "one at least of the two breads is refused"

(One will give the round decimal approximations to  $10^{-3}$  close to these probabilities while using for calculations, the approximate result obtained with the preceding question.)

- 2) Determination of binominal distribution and approximation by a normal law the company produced, before packing, 10.000 breads per week. One notes  $Y$  the
- 3) random variable measuring the number of refused breads, in the one week production.
- a) To explain why  $Y$  follows the binomial distribution. Give the parameters of this law calculate the average and the standard deviation of  $Y$  (one will round the last two results with the whole numbers closest)
- b) One replaces the law  $Y$  by the normal which constitutes a satisfactory approximation of it.

Justify and give the parameters of those?

Calculate, by using this approximation, probability of the event

$$"650 \leq Y \leq 700" \text{ (One will round with the result with } 10^{-2} \text{)}$$