CORRECTION OF 2013 SESSION

1) Determination of x and y- intercepts of the function $f(x) = \frac{x^2 - 5x + 4}{x^2 - 3}$

It is easily clear to see that $f(0) = -\frac{4}{3}$ and $f(1) = f(4) = 0$ therefore we can just conclude the x and y- intercepts are 1, 4 and -4/3 thus **1 – B**

2). The function $y = \sqrt[3]{x}$ increase and symmetric with respect to origin because it is the reciprocal of the function $f(x) = x^3$ hence **2 – B**

3) D is the function of the limit of the CAUCHY sequence therefore **3 – D**

4) $f(x) = x^3 + 2x + 2, x \in \mathbb{R}$ the zero of the function f is the real x_0 such that $-1 \leq x_0 \leq 0$

We know that f(-1) = -1 and f(0) = 2 and according to the intermediate value theorem:

$f(0) \times f(-1) = -2 < 0$. Hence $x_0 \in [-1,2]$ therefore **4 – B**

5) $y(t) = Me^{\frac{1}{3}t\ln(10)}$ because $\begin{cases} y(0) = M \\ y(3) = Me^{\frac{1}{3}(3)\ln(10)} = Me^{\ln(10)} = 10M \end{cases}$

Therefore **5 – D**

6) Knowing that $U_0 = 1million = 10^6$

At the end of the first year we have $U_1 = U_0 + 5\%U_0 = 1.05U_0$.

At the end of the second year we have

$U_2 = (U_1 + 10^6) + 5\%(U_1 + 10^6) = 1.05\left(U_1 + 10^6\right)$

We can the generalize that $U_{n+1} = 1.05\left(U_n + 10^6\right)$ the calculation of

U_{25}=50.133.250

Therefore **6 – C**

$$s_n = \sum_{n=1}^{\infty} \frac{3+5n}{4} x^{n+1} a_n = \frac{3+5n}{4}; a_{n+1} = \frac{3+5(n+1)}{4}$$

$$l = \lim_{x\to\infty} \frac{a_{n+1}}{a_n} = \lim_{x\to\infty} = \frac{5(n+1)+3}{5n+4} \Rightarrow l = 1$$

$R = \frac{1}{l} = 1$ Hence S_n converge $if\,|x| \prec R \Rightarrow |x| \prec 1 \Rightarrow -1 \prec x \prec 1$

Therefore **7 – D**

If $x_1 \geq 0 \Rightarrow x_n \geq 0$ the n we have $\frac{1}{2}\left(x_n + \frac{a}{x_n}\right) \Rightarrow x_{n+1} \geq 0$

(X_n) Is the positive terms sequence and $x_{n+1} - x_n = \frac{1}{2}\left(x_n + \frac{a}{x_n}\right) - x_n = \frac{1}{2}\left(\frac{a}{x_n} - x_n\right)$ by

resolution of the equation $X_{n+1} = X_n$ we can easily have

$$x_n = \frac{1}{2}\left(x_n + \frac{a}{x_n}\right) \Rightarrow 2x_n = \left(x_n + \frac{a}{x_n}\right) \Rightarrow x_n = \frac{a}{x_n} \Rightarrow x_n = \pm\sqrt{a} \lim_{x \to \infty} x_n = \sqrt{a}$$

Therefore **8 – B**

9) $g(x) = x^2 f\left|\cos\frac{2\pi}{3}\right| \Rightarrow g`(x) = 2xf\left|\cos\frac{2\pi}{3}\right| - \frac{2\pi}{3}x^2 \sin\frac{2\pi}{3} f`\left|\cos\frac{2\pi}{3}\right|$

For X=1 $\Rightarrow g`(1) = 2f\left(\frac{1}{2}\right) - \frac{\pi}{3}\frac{\sqrt{3}}{2} f`\left(\frac{1}{2}\right) \Rightarrow g`(1) = 2 + \frac{\sqrt{3}}{6}\pi$

9 – B

10) $f(x) = \frac{x}{x^2+1} \Rightarrow f`(x) = \frac{x^2 - 2x(x)}{(x^2+1)^2}$

X	$-\infty$ -1	1 $+\infty$	
$f(x)$	−		−

Therefore

10– C

11) $(x^3 + a^2 y)dx + (4x + y^3)dy = 0 \Rightarrow a = -2; 2$

11– C

12) the general solution of the equation $y''-2y'+2y=0$ is on the form $y=(a\cos x+b\sin x)e^x = a\cos xe^x + b\sin xe^x$ where a and b are real numbers.

Here the two linearly functions for that solution are: $y_1 = a\sin xe^x$ and $y_2 = b\cos xe^x$ and the wronskian (W) of those two solutions is:

$$W(y_1, y_2) = \begin{vmatrix} y_1 y_2 \\ y_1' y_2' \end{vmatrix} = \begin{vmatrix} (sinx)e^x & (cosx)e^x \\ (cosx + sinx)e^x & (cosx - sinx)e^x \end{vmatrix} = -e^{2x}$$

For x = ln2, W = $-e^{2ln2} = -e^{ln4} = -4$ therefore question 12 has no good answer

19) $(1,1,6);(2,-1,2);(0,1,4)$ Verify the equation $z=2X+3Y+1$

Therefore**<u>19– A</u>**

20) Since $\left|\frac{-1}{9}\right| = \left|\frac{1}{9}\right| \prec 1$ one has**<u>20 – A</u>**