www.tousles concours.info

www.tousles concours.info

CORRECTION OF 2012 SESSION

EXERCISE 1

1) Let determine whether the following series converges or not.

We have

$$\sum_{n=1}^{\infty}\frac{1}{2^n+1} \Rightarrow u_n = \frac{1}{2^n+1} \Rightarrow \lim_{x\to\infty} u_n = \lim_{x\to\infty}\left(\frac{1}{2^n+1}\right) = 0$$ thus the series $\sum_{n=1}^{\infty}\frac{1}{2^n+1}$ converges.

2) $\sum_{n=1}^{\infty}\frac{3n+1}{n^3+1} \Rightarrow u_n=\frac{3n+1}{n^3+1} \Rightarrow \lim_{x\to\infty} u_n=\lim_{x\to\infty}\frac{3n+1}{n^3+1}=\lim_{x\to\infty}\frac{3n}{n^3}=\lim_{x\to\infty}\frac{3}{n^2}$ according to RIEMAN

criteria ,one has $\frac{1}{n^\alpha} \Rightarrow \begin{cases} \alpha\le 1 \Rightarrow diverges \\ \alpha \ge 1 \Rightarrow converges \end{cases}$

Since $\alpha=2\succ 1$ the series $\sum_{n=1}^{\infty}\frac{3n+1}{n^3+1}$ converges

3) $\sum_{n=1}^{\infty}\frac{1}{\ln(n)} \Rightarrow u_n=\frac{1}{\ln(n)}$

The function u_n is positive $\forall n\in \square^*$ and decreasing on $[2,+\infty[$ and

$u'(n)=\frac{-1}{n\ln(n)}$ but according to **BERTRAND criteria** $\frac{-1}{n^\alpha(\ln(n))^\beta} \Rightarrow \alpha=1, \beta\succ 1$ the series converges.

4) $\sum_{n=1}^{\infty}\frac{99^n}{n!} \Rightarrow u_n=\frac{99^n}{n!}$ by applying **D'ALEMBERT criteria**

We have $\frac{u_{n+1}}{u_n}=l \Rightarrow \begin{cases} l\ge 1 \Rightarrow diverges \\ l\prec 1 \Rightarrow converges \end{cases}$ We know that $u_{n+1}=\frac{99^{n+1}}{(n+1)!}=\frac{99^n}{(n+1)!}\times 99$ and

$\frac{u_{n+1}}{u_n}=\frac{\left(\frac{99^n}{(n+1)!}\times 99\right)}{\left(\frac{99^n}{n!}\right)}=\frac{99n!}{(n+1)n!}=\frac{99}{(n+1)}$ but $\frac{1}{(n+1)} \square \frac{1}{n}$ according to **RIEMANN**

then $\frac{u_{n+1}}{u_n}=\frac{99}{n} \Rightarrow diverges$

5) $\sum_{n=1}^{\infty}\frac{n^5}{2^n}=\frac{1}{2^1}(1)^5+\frac{1}{2^2}(2)^5+\frac{1}{2^3}(3)^5+\cdots$

Here the series is progressing in increasing to certain real or values thus the series $\sum_{n=1}^{\infty}\frac{n^5}{2^n}$ diverges

EXERCISE 2

www.tousles concours.info

Let determine the derivatives of the following functions knowing that f is derivable.

www.touslesconcours.info

We will always remember that $(fog)'(x) = g'(x) \times (f'og)(x)$

1) We have g(x) = f(3x)

$$g'(x) = f'(3x) = (fo(3x))' = (3x)' \times f'o(3x) = 3f'(3x)$$

2) $\left(f(x^2)\right)' = \left(x^2\right)' f'\left(x^2\right) = 2xf'\left(x^2\right)$

EXERCISE 3

1) Let verify if the following differential equations is exact.

$$\left(2x + \sin y - ye^{-x}\right)dx + \left(x\cos y + \cos y + e^{-x}\right)dy = 0$$

Let $f(x,y)dxdy = 0$ and

$f(x,y) = f(x_0, y_0) + (x - x_0)\dfrac{\partial f}{\partial x} + (y - y_0)\dfrac{\partial f}{\partial y} = 0 \Rightarrow \dfrac{\partial f}{\partial x} = -\dfrac{\partial f}{\partial y}$ because it verify the complete condition of integrate.

NB: there are many others method to prove the exactitude of this equation.

Let now determine the curves solutions of this equation.

$$\left(2x + \sin y - ye^{-x}\right)dx + \left(x\cos y + \cos y + e^{-x}\right)dy = 0 \Rightarrow \left(2x + \sin y - ye^{-x}\right)dx = -\left(x\cos y + \cos y + e^{-x}\right)dy$$

$$\Rightarrow x^2 + x\sin y + ye^{-x} + c_1 = x\sin y - \sin y - ye^{-x} + c_2$$

$$\Rightarrow x^2 + \sin y + 2ye^{-x} = c_T$$

Which $C_T(x,y) = C(x,y) = x^2 + \sin y + 2ye^{-x}$ is the characteristic curves function.

2) Let solve the differential equation $\dfrac{\partial y}{\partial x} + xy = x^3$

www.touslesconcours.info

$$\frac{\partial y}{\partial x}+xy=x^3 \Rightarrow y'+xy=x^3 \text{, let } u=e^{\int xdx}=e^{\frac{x^2}{2}} \Rightarrow u'=xu$$

By multiplying the equation by u we obtain:

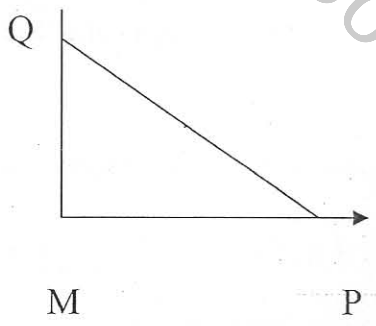
$$uy'+uxy=ux^3 \Rightarrow (uy)'=ux^3 \Rightarrow y=\frac{1}{u}\int x^3 e^{\frac{x^2}{2}}dx$$

After integrating the line above by part we obtain $y=\left(x^2-2+ke^{-\frac{x^2}{2}}\right)$ where k is a real number.

www.tousleconcours.info

EXERCISE 4

Let evaluate $\iint_T (x,y)\,dA$ on M (0, 0); P (1, 0); Q (1, 1)



We have $\int_0^1 \int_0^1 (x,y)d\left(\frac{1}{2}xy\right) = \frac{1}{2}\int_0^1 \int_0^1 (x,y)d(xy) = \frac{1}{2}\left[\frac{1}{2}x^2\right]_0^1 \left[\frac{1}{2}y^2\right]_0^1 = \frac{1}{8}$

EXERCISE 5

1) Let find

$$\int_0^3 f(x)dx where f(x) = \begin{cases} \sqrt{1-x^2} \Rightarrow 0 \le x \le 1 \\ 2 \Rightarrow 1 \le x \le 2 \\ x-2 \Rightarrow 2 \le x \le 3 \end{cases}$$

By applying the integral sum of functions, we obtain according to CHASLES
www.touslesconcours.info

theorem: $\int_0^3 f(x)dx = \int_0^1 \sqrt{1-x^2}\,dx + \int_1^2 2dx + \int_2^3 (x-2)\,dx$

At this level it is clear that everybody can easily came out with the result

2) Let find the total area A laying between the curves y = sinx and cosx from x = 0 to x = 2π

Since $0 \leq x \leq 2\pi, one can simplify to find the area by$

$A = \int_0^{2\pi} \sin x dx + \int_0^{2\pi} \cos x dx$ and we also know the formula that represent the average value of a periodic signal, then we can easily obtain A = 0 area unit

www.touslesconcours.info

CORRECTION OF 2012 SESSION