CORRECTION OF 2010 SESSION

EXERCISE 1

1) Let $f(x) = ax^3 + bx + c$a periodic function of period T = 2π such that

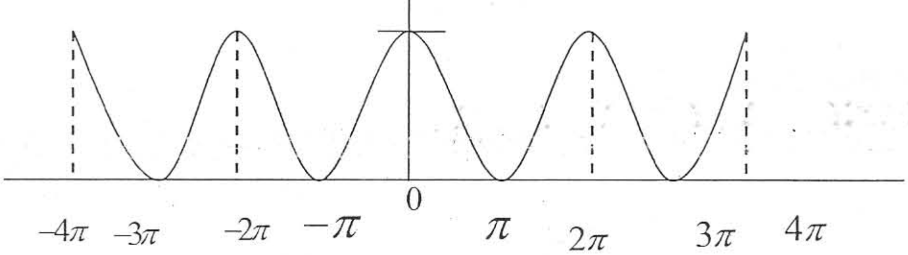
$f(0) = f(2\pi) = \pi$ and $f(\pi) = 0$

$$\begin{cases} f(0) = \pi \Rightarrow a(0)^2 + b(0) + c = \pi \\ f(\pi) = 0 \Rightarrow a(\pi)^2 + b(\pi) + c = 0 \\ f(2\pi) = \pi \Rightarrow a(2\pi)^2 + b(2\pi) + c = \pi \end{cases}$$

After the transformation we easily obtain $a = \frac{1}{\pi}; b = -2; c = \pi$ then

$$f(x) = \frac{1}{\pi}x^2 - 2x + \pi$$

2) Let draw the curve of f on the interval $[-4\pi, 4\pi]$



3- a) let calculate the average value of f on one period

$$a_0 = \frac{2}{T}\int_0^{\frac{T}{2}} f(x)dx \Rightarrow f(x) = \frac{2}{2\pi}\int_0^{\pi}\left(\frac{1}{\pi}x^2 - 2x + \pi\right)dx = \frac{\pi}{3}$$

b)- Let determine $S(f)(x)$ the Fourier sum of f

We have $S(x) = a_0 + \sum_{n\to 0}^{\infty} a_n\cos(n\omega t) + b_n\sin(n\omega t)$ but $a_0 = \frac{\pi}{3}$ and

$a_n = \frac{2}{T}\int_0^{\frac{T}{2}} f(x)\cos(n\omega t)dt \Rightarrow a_n = \frac{4}{2\pi}\int_0^{2\pi}\left(\frac{1}{\pi}x^2 - 2x + \pi\right)\cos(n\omega t)dt$ by integrating twice

a_n we obtain $a_n = -\frac{4}{\pi n^2}$.

Therefore the expression of the Fourier's series is given by know that

c) Let explain why the sum converges and write the corresponding equality.

We know that in Fourier's series the sum converges if and only if

$\lim_{x\to\infty}(s(n)) = \Delta(n) = 0$.

Now let write the corresponding equality $\lim(s(n)) = A(t) = F(f(t))$ the using of the DIRICHEL criteria one has $(s(n)) = F(f(t)) = \frac{\pi}{3} + \sum_{n=1}^{\infty} -\frac{4}{\pi}\cos(n\omega t) = 0$ since n = 1 moreover $f(\pi) = 0$

4 – a) let show that the sum of general term $\frac{(-1)^n}{n^2}$ converges

By applying the RIEMAN criteria, $\frac{1}{n^{\alpha}} \begin{cases} diverge \Rightarrow \alpha \leq 1 \\ converge \Rightarrow \alpha \geq 1 \end{cases}$

By observation $\alpha = 2 \geq 1$,hence $U_n = \frac{(-1)^n}{n^2}$ converge

b) determination of the sum: by using question 3 – c) one has

$\frac{\pi}{3} + \sum_{n=1}^{\infty} -\frac{4}{n^2\pi}\cos(nt) = 0 \Rightarrow -\sum_{n=1}^{\infty}\frac{4}{n^2\pi}\cos(nt) = -\frac{\pi}{3}$ where $\cos nt = (-1)^n, \forall n$

$$\Rightarrow \sum_{n=1}^{\infty}\frac{4}{n^2\pi}(-1)^n = \frac{\pi}{3} \Rightarrow \frac{\pi}{3} + \sum_{n=1}^{\infty} -\frac{4}{n^2\pi}\cos(nt) = 0 \Rightarrow \sum_{n=1}^{\infty}\frac{(-1)^n}{n^2} = \frac{\pi^2}{12}$$