Mathematics Entrance Examination For All General Education

Cycle 1, Campus Bambili

Coefficient: 4 **Duration: 4 hours**

Exercise (4 marks)

One poses $I_n = \int_0^{\frac{\pi}{2}} \cos^n(t)dt$ for any $n \geq 1$.

1. Calculate I_0, I_1 and I_2.
2. Show that for all $n \geq 2$, one has $nI_n = (n-1)I_{n-2}$.
3. Deduce the value of I_n for any $n \geq 1$.

Exercise (4 marks) 1. Study the continuity of the following function:

$$f(x) = \begin{cases} x^2 & \text{if} \quad x \leq 0, \\ x & \text{if} \quad 0 < x < 2, \\ 4-x & \text{if} \quad x \geq 2. \end{cases}$$

2. Show the function g is continuous and differentiable and its derivative is continuous

$$g(x) = \begin{cases} e^{\frac{1}{x}} & \text{if} \quad x < 0, \\ 0 & \text{if} \quad x = 0, \\ \cos(x) - 1 & \text{if} \quad x > 0. \end{cases}$$

Exercise (3 marks)

A discrete random variable X has the following probability distribution.

$X = x$	0	1	2	3	4
$P(X = x)$	0.12	p	0.4	q	0.08

Given that $E(X) = 2$, find

1. The value of p and q.
2. $\text{Var}(X)$
3. The mean and variance of $Y = 5X + 7$.

Exercise (9 marks)

Part I: (6 marks) Let f be the function defined on $\mathbb{R}$ by $f(x) = \frac{3(x-1)^2}{3x^2+1}$ and let (C) its curve representative in the plane brought back to an orthogonal reference mark of unit 1 cm.

1. Show that there exists a single triplet (a, b, c), that one will determine, such that for any real x: $f(x) = ax + b + \frac{cx}{3x^2+1}$.
2. Determine the limits of f in $+\infty$ and in $-\infty$.
3. Show that f is differentiable and calculate its derivative.
4. Draws the table of variation of f.
5. Show that the curve (C) has an asymptote oblique the line (D) of equation $y = x - 3$.
6. Study the relative positions of (C) and (D).
7. Give the equation of the tangent (T) to (C) at the points of x-coordinate 0. Trace (T), (C) and (D).
8. Show that the curve (C) has a centre of symmetry.
9. Show that the equation $f(x) = 1$ has a single solution in $\mathbb{R}$. One notes α solution.
10. Give the approximative value of α to 10^{-2} near by excess.

Part II: (3 marks)

One considers the function g defined on $\mathbb{R}$ by $g(x) = \frac{3(\sin(x)-1)}{3\sin^2(x)+1}$.

1. Show that g is differentiable on $\mathbb{R}$ and calculate $g'(x)$.
2. Draw the table of variation of g on $[-\pi; \pi]$.
3. Plot on a new drawing the curve representative of g.