Entrance Examination, July 2011 Cycle: 1 Campus: Bambili

Series: Civil and Forestry Technical Education

Topic: Mathematics Duration: 4 h Coefficient: 2 BAC F & TECH (minor)

Exercise (7 marks)

A- Let be the function defined on $]0;+\infty[$ by $f(x) = x - 2 + \frac{1}{2}\ln(x)$.

(a) i. Calculate the limits of f in 0 and in $+\infty$.

ii. Calculate $f'(x)$ and give the variation sense of f.

(b) i. Show that the equation $f(x) = 0$ has a unique solution denoted α on $]0;+\infty[$. Give the approximative value of α at 10^{-2} near.

ii. Study the sign of $f(x)$.

B- Let g be the function defined on $[0;+\infty[$ by $g(x) = -\frac{7}{8}x^2 + x - \frac{1}{4}x^2\ln(x)$ for all $x > 0$ and $g(0) = 0$.

(a) i. Study the continuity and the differentiation of g in 0.

ii. Determine the limit of g in $+\infty$.

(b) Let's consider g' the derivative of the function g. For all $x > 0$, calculate $g'(x)$, and verify that $g'(x) = xf(\frac{1}{x})$.

i. Deduce the sign of $g'(x)$, draw the variation table of g.

ii. Give the equations of the tangents to the curve representative (C) of g at the points of x-coordinates 0 and 1. Plot (C) and the previous tangent.

Exercise (4 marks)

Evaluate

1. $\int_0^{\sqrt{3}} x^2 \arctan(x)dx$
2. $\int_0^1 \frac{1}{(1+x^2)^2}dx$

Exercise (3 marks)

Solve the following differential equation : $y' + y = 2\cos(x) + (x+1)e^{-x}$.

Exercise (6 marks)

Let's consider the numerical sequence (u_n) defined by $u_0 = 0$ and for all $n \in \mathbb{N}$, $v_n = \frac{2}{u_n}$ and $u_{n+1} = \frac{u_n + v_n}{2}$.

1. Calculate v_0, u_1,v_1,u_2 et v_2. Give the results in the form of non-reducible fraction.

2. Show that (u_n) and (v_n) are bounded above by 2 and bounded below by 1.

3. Show that for all $n \in \mathbb{N}$, $u_{n+1} - v_{n+1} = \frac{(u_n - v_n)^2}{2(u_n + v_n)}$.

4. Show that for all $n \in \mathbb{N}$, $u_n \geq v_n$.

5. Show that (u_n) is decreasing and (v_n) is increasing.

6. Show that for all $n \in \mathbb{N}$, $u_n - v_n \leq 1$. Deduce that $(u_n - v_n)^2 \leq u_n - v_n$.

7. (a) Show that for all $n \in \mathbb{N}$, $u_{n+1} - v_{n+1} \leq \frac{1}{4}(u_n - v_n)$.

   (b) Show that for all $n \in \mathbb{N}$, $|u_n - v_n| \leq \frac{1}{4^n}$.

8. Show that (u_n) et (v_n) converge to the same limit.