Test of specialty: Civil and Forestry Engineering

First year entrance examination. Option: "F4, MEB"

Session: 2009 Duration: 3H

Exercise (5 marks)

The complex plane is reported to direct orthonormal line $(O, \vec{u}, \vec{v})$; graphic unit: 2 cm.

1. Draw the circles whose center is O the radius 1 and 2. Indicate points A, B and D with respective affixes $\sqrt{3}+i$, $\sqrt{3}-i$ and $-\frac{1}{2}+\frac{\sqrt{3}}{2}i$.

2. We consider rotation R with center O and angle $\frac{\pi}{3}$ ant translation T vector with affixes 1.

   (a) Determine affixes $Z_{A'}$ and $Z_{B'}$ of points A' and B', representative image of points A and B by rotation R.

   (b) Determine affixes $Z_{D'}$ of point D', image of point D by the translation T.

   (c) Indicate point A', B' and D'.

3. Determine an argument of the complex number $\frac{Z_{A'}-Z_{B'}}{Z_{D'}}$. Prove that line (OD') is the median of triangle $OA'B'$.

Exercise (5 marks)

We consider the differential equation $y''+5y'=0$ (E).

1. Prove that the function f, is solution of (E) if and only if function $F=f'$ is solution of $y'+5y=0$ (E_1).

2. Resolve the differential equation (E).

3. For any real x, we take it that $g(x)=a\cos(x)+b\sin(x)$ where a and b are reals. Determine real a and b in such a way that g checks the differential equation

   $y''+5y'=26\cos(x)$ (E').

4. Prove that f is solution of the differential equation (E').

5. Determine all the solutions of the differential equation (E').

6. Determine the solution of (E') checking $f(0)=0$ and $f'(0)=0$.

Exercise (6 marks)

We consider the defined function for any real x not nil by $f(x) = \frac{2e^x+3}{e^x-1}$. The curve representative of function f in an orthonormal line $(O,\vec{i},\vec{j})$ is call C_f.

1. Study the limits of f in $+\infty$ and $-\infty$.
2. Study the limits of f, when x turns towards 0 by superior values or by inferior values.
3. Deduce from the study the asymptotes s of C_f.
4. Calcule the derivative of f. Study the variation of f.
5. Prove that point $\Omega(0;-\frac{1}{2})$ is the center of symmetry of curve C_f.

Exercise (4 marks)

The exercise has 4 affirmations. Indicate for each of them if is true or false and justify your answer given that the function is defined by $f(x) = \ln(\frac{2x+1}{x-1})$.

1. f is defined on $]1;+\infty[$.
2. $f'(x) = -\frac{1}{(x-2)^2}\ln(\frac{2x+1}{x-1})$.
3. Line $x = 1$ is the asymptote to the representative curve of f.
4. The representative curve of f admits a horizontal asymptote.