Solution of first year entrance in the first cycle

Series: C, D, E & GCE AL (2011-2012)

Exercise

Let us study the movement in the interval $I = [0;+\infty[$ on which the horary low is:

$$\varphi(t) = \cos\frac{\pi}{4}t + \sin\frac{\pi}{4}t + 1$$

1. Nature of the movement.

   Here the movement of $\varphi(t)$ is sinusoidal.

2. We give his magnitude and its horde.

   Magnitude

$$\varphi(t) = \cos\frac{\pi}{4}t + \sin\frac{\pi}{4}t + 1.$$
$$\varphi'(t) = -\frac{\pi}{4}\sin\frac{\pi}{4}t + \frac{\pi}{4}\cos\frac{\pi}{4}t$$

$$\begin{aligned}
\varphi'(t) = 0 \quad &\Longleftrightarrow \quad -\frac{\pi}{4}\sin\frac{\pi}{4}t + \frac{\pi}{4}\cos\frac{\pi}{4}t = 0 \\
&\Longleftrightarrow \quad -\sin\frac{\pi}{4}t + \cos\frac{\pi}{4}t \\
&\Longleftrightarrow \quad \sin\frac{\pi}{4}t = \cos\frac{\pi}{4}t \\
&\Longleftrightarrow \quad \cos\frac{\pi}{4}t = \cos(\frac{\pi}{4}t - \frac{\pi}{2}) \\
&\Longleftrightarrow \quad \begin{cases} \frac{\pi}{4}t = \frac{\pi}{4}t - \frac{\pi}{2} + 2k\pi \\ \frac{\pi}{4}t = -\frac{\pi}{4}t + \frac{\pi}{2} + 2k\pi \quad k \in \mathbb{Z} \end{cases} \\
&\Longleftrightarrow \quad \frac{\pi}{2}t = \frac{\pi}{2} + 2k\pi (k \in \mathbb{Z}) \\
&\Longleftrightarrow \quad t = 1 + 4k, (k \in \mathbb{Z})
\end{aligned}$$

   For $k = 0$, $t = 1$

   $\varphi(1) = \cos\frac{\pi}{4} + \sin\frac{\pi}{4} + 1 = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} + 1 = \sqrt{2} + 1.$

   Therefore, the magnitude is $\sqrt{2} + 1$.

   Horde

   Its horde is $\frac{\pi}{4} rad/s$.

3. We define its center and deduce the reference $(O, \vec{u})$. Therefore its center is $C = (0, 1.$
   $\varphi(1) - 1 = 1, (O, \vec{u})$ is the origin of the date.

4. We calculate its period and its phase.

$$T = \frac{2\pi}{\omega}$$

NA: $T = \frac{2\pi}{\frac{\pi}{4}} = 2\pi \times \frac{4}{\pi}$. $\boxed{\text{T=8 s}}$

$$\begin{aligned} \cos\frac{\pi}{4}t + \sin\frac{\pi}{4}t &= \sqrt{2}(\frac{\sqrt{2}}{2}\cos\frac{\pi}{4}t + \frac{\sqrt{2}}{2}\sin\frac{\pi}{4}t) \\ &= \sqrt{2}(\cos\frac{\pi}{4}\cos\frac{\pi}{4}t + \sin\frac{\pi}{4}\sin\frac{\pi}{4}t) \\ &= \sqrt{2}\cos(\frac{\pi}{4}t - \frac{\pi}{4}) \end{aligned}$$

Therefore its is $\boxed{-\frac{\pi}{4}}$

let be $(C,\vec{u})$ the new reference defined so that $x_1(t) = A\cos(\omega t + \varphi_0)$.

5. We determine A, ω and φ_0.

   In this reference, $x_1(t) = \sqrt{2}\cos(\omega t + \varphi_0)$. So $\boxed{A = \sqrt{2}, \quad \omega = \frac{\pi}{4} rad/s, \quad \varphi_0 = -\frac{\pi}{4} rad}$

   We now study the position of the point M on the reference $(O,\vec{u})$ corresponding to M_0 in the reference $(C,\vec{u})$.

6. Let's calculate the position $x_1(t)$, its algebraic motion $v(0)$ and its acceleration $\gamma(0)$.

$$x_1(0) = \sqrt{2}\cos(\frac{\pi}{4}.0 - \frac{\pi}{4}) = \sqrt{2}\cos(-\frac{\pi}{4}) = 1$$

$$x_1'(t) = v(t) = (\sqrt{2}\cos(\frac{\pi}{4}t - \frac{\pi}{4}))' = -\sqrt{2}\frac{\pi}{4}\sin(\frac{\pi}{4}t - \frac{\pi}{4})$$

$$v(t) = -\sqrt{2}\frac{\pi}{4\sin(\frac{\pi}{4}t - \frac{\pi}{4})}$$

$$v(0) = -\sqrt{2}\frac{\pi}{4}\sin(\frac{\pi}{4}.0 - \frac{\pi}{4}) = -\sqrt{2}\frac{\pi}{4}\sin(-\frac{\pi}{4}) = -\sqrt{2}\frac{\pi}{4}.(-\frac{\sqrt{2}}{2}); \quad v(0) = \frac{\pi}{4} rad/s$$

$$x''(t) = v'(t) = \gamma(t) = -\sqrt{2}\frac{\pi}{4}\sin(\frac{\pi}{4}t - \frac{\pi}{4})' = -\frac{\pi^2}{16}\sqrt{2}\cos(\frac{\pi}{4}t - \frac{\pi}{4})$$

$$\gamma(0) = -\frac{\pi^2}{16}\sqrt{2}\cos(-\frac{\pi}{4}) = -\frac{\pi^2}{16} rad/s^2$$

7. We calculate the product $\boxed{v(0).\gamma(0) = \frac{\pi}{4}(-\frac{\pi^2}{16}) = -\frac{\pi^3}{64}}$

   Since $v(0).\gamma(0) < 0$, the movement is retarded at $t = 0s$.

8. We reduce the direction of movement at the initial time.

$$x(0) = 1, v(0) = \frac{\pi}{4} rad/s$$

   Therefore the movement is in the positive direction.

9. We determine the points in which the movement will be equal to zero in the interval $[0;8]$. $x_1(t) = \sqrt{2}\cos(\frac{\pi}{4}t - \frac{\pi}{4})$

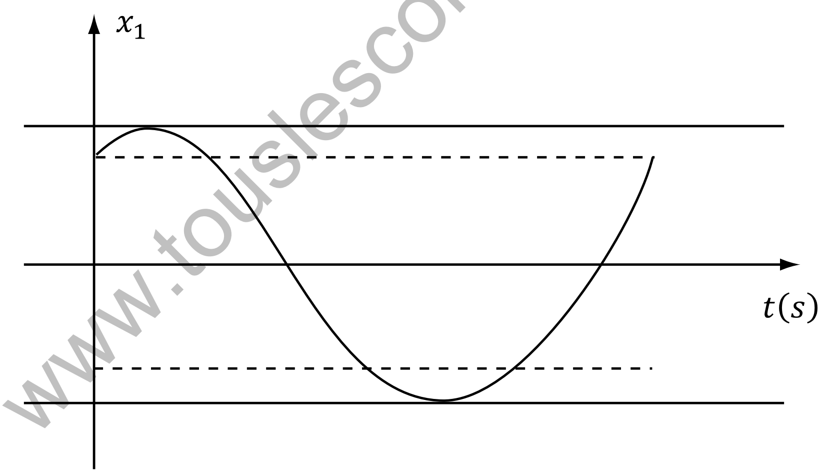
$$\begin{aligned} x_1(t) = 0 &\iff \sqrt{2}\cos(\frac{\pi}{4}t - \frac{\pi}{4}) = 0 \\ &\iff \cos(\frac{\pi}{4}t - \frac{\pi}{4})0 \\ &\iff \cos(\frac{\pi}{4}t - \frac{\pi}{4}) = \cos\frac{\pi}{2} \\ &\iff \frac{\pi}{4}t - \frac{\pi}{4} = \frac{\pi}{2} + k\pi, (k \in \mathbb{Z}) \\ &\iff \frac{\pi}{4}t = \frac{3\pi}{4} + k\pi \\ &\iff t = 3 + 4k, (k \in \mathbb{Z}) \end{aligned}$$

For $k = 0, t = 3s; k = 1, t = 7s$, so we have $\boxed{(3,0); (7,0)}$

10. We calculate $x_1(5)$

$$x_1(5) = \sqrt{2}\cos[\frac{\pi}{4}(5) - \frac{\pi}{4}] = -\sqrt{2}$$

11. We draw the diagram of the function within its period.



Exercise

Let's X the variation. We have the followings events:

A: "The cylinder is accepted";

B: "The cylinder undergoes a correction"

1. The cylinder is accepted if the variation is inferior to 1 or if one proceeds to a correction when the variation lies between 1 and 2 it is accepted.

$$P(A) = P\{X < 1\} + P(A \cap B) = P\{X < 1\} + P(A/B) \times P(B)$$

where A/B is the event "A knowing B".

$$P\{X<1\} = \int_0^1 e^{-\gamma t}dt = [-e^{-\gamma t}]_0^1 = 1-e^{-\gamma} = 1-e^{-1.5} = 0.776$$

$$P\{A\cap B\} = P(A\cap B)\times P(B) = 0.8\times P(B)$$

$$P(B) = P\{1\leq X\leq 2\} = P\{X\leq 2\} - P\{X\leq 1\}$$

$$P\{X\leq 2\} = \int_0^2 e^{-\gamma t}dt = [-e^{-\gamma t}]_0^2 = 1-e^{-2\gamma} = 1-e^{-3} = 0.950$$

$$P(B) = 0.950 - 0.776 = 0.174$$

$$P(A\cap B) = 0.8\times 0.174 = 0.1392 \text{ from where}$$

$$P(A) = 0.776 + 0.1392 = 0.152$$

$$\boxed{P(B/A) = \tfrac{P(A\cap B)}{P(A)} = \tfrac{0.1392}{0.915} = 0.152}$$

2. (a) Probability that at least a cylinder is refused.

$$\boxed{[P(A)]^{10} = [0.915]^{10} = 0.411}$$

   (b) Probability that at least a cylinder is refused.

   Let's E the event "at least a cylinder is refused". The contrary event is $\overline{E}$: "the ten cylinders are accepted".

$$\boxed{P(E) = 1-[P(A)]^{10} = 1-0.411 = 0.589}$$