Correction of entrance examination into year I; 1st cycle HTTTC Bambili

Department: Fundamental sciences 2012 – 2013

Option: All technical/industrial series

Exercise

Given the complex numbers $z_1 = 3(-\frac{1}{2} + i\frac{\sqrt{3}}{2})$ and $z_2 = 3(\frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2})$.

1. Let us give the trigonometric form of:

$$|z_1| = \frac{3}{2}\sqrt{(1)^2 + (\sqrt{3})^2} = 3$$

$$\arg(z_1) = arctg(-\frac{\sqrt{3}}{2} \times 2) = \frac{2\pi}{3}$$

Therefore,

$$\boxed{z_1 = 3(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3})}$$

$$|z_2| = \frac{3}{2}\sqrt{(\sqrt{2})^2 + (\sqrt{2})^2} = 3$$

$$\arg(z_2) = arctg(\frac{\sqrt{2}}{2} \times \frac{2}{\sqrt{2}}) = \frac{2\pi}{3} = \frac{\pi}{4}$$

Therefore,

$$\boxed{z_2 = 3(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4})}$$

$$\frac{z_1}{z_2} = \frac{3e^{i\frac{2\pi}{3}}}{3e^{i\frac{\pi}{4}}} = e^{i\frac{5\pi}{12}}$$

Therefore,

$$\boxed{\frac{z_1}{z_2} = \cos\frac{5\pi}{12} + i\sin\frac{5\pi}{12}}$$

2. Let us prove that for all interger n z^{12n} is a real number.

We know from the above question that $z = \cos\frac{7\pi}{12} - i\sin\frac{7\pi}{12}$

$$\begin{aligned} z^{12n} &= (\cos\frac{7\pi}{12} - i\sin\frac{7\pi}{12})^{12n} \\ &= \cos\frac{7\pi}{12}(12n) - i\sin\frac{7\pi}{12}(12n) \quad \text{(Moivre theorem)} \end{aligned}$$

Thus, for all n interger $z^{12n} = \cos(7\pi n)$ $(\sin 7\pi n = 0)$

Let's give the exact values of $\cos\frac{5\pi}{12}$ and $\sin\frac{5\pi}{12}$.

From the preceding questions we have: $z = \frac{z_1}{z_2} = \cos\frac{5\pi}{12} - i\sin\frac{5\pi}{12}$.

But $\cos 2(\frac{5\pi}{12}) = 2\cos^2(\frac{5\pi}{12}) - 1$, thus $\cos(\frac{5\pi}{6}) = 2\cos^2(\frac{5\pi}{12}) - 1$.

Which bring us to: $\cos\frac{5\pi}{12} = \frac{\sqrt{2-\sqrt{3}}}{2}$ (a)

One the other side, we have: $\sin\frac{5\pi}{12} = \frac{\sqrt{2+\sqrt{3}}}{2}$ (b)

Thus from (a) and (b), we obtain:

$$\boxed{\cos\frac{5\pi}{12} = \frac{\sqrt{2-\sqrt{3}}}{2} \text{ and } \sin\frac{5\pi}{12} = \frac{\sqrt{2+\sqrt{3}}}{2}}$$

Exercise

Caractestic equation $4r^2 - 4r + 1 = 0$.

The calculation of the discriminant yield to $r = \frac{1}{2}$. $f(x) = e^{\frac{x}{2}}(Ax+B)$ where A, B belonging to $\mathbb{R}$.

Let us determine the constants A and B. $f(0) = B = 4$ so $B = 4$

$$f'(x) = \frac{1}{2}e^{\frac{x}{2}}(Ax+B) + Ae^{\frac{x}{2}} = [\frac{1}{2}(Ax+B) + A]e^{\frac{x}{2}}$$

But $f'(2) = 0$; $B = 4$. That is $f'(2) = \frac{1}{2}(2A+4) + A = 0$ thus $A = -1$.

Hence,

$$\boxed{f(x) = e^{\frac{x}{2}}(4-x)}$$

Exercise

Let us solve for (x, y) in the set of real numbers the system of equation $\begin{cases} x^2 + y^2 = 145 \\ \ln x + \ln y = \ln 72 \end{cases}$

This implies that $\begin{cases} x^2 + y^2 = 145... \ (1) \\ xy = 72... \ (2) \end{cases}$ (2) implies $x = \frac{72}{y}$...(3).

(3) into (1): $(\frac{72}{y})^2 + y^2 = 145$ i.e $y^4 - 145y^2 + 72^2 = 0$ i.e $y^4 - 145y^2 + 5184 = 0$.

That $(y^2 - 64)(y^2 - 81) = 0$ which yields to $y = 8$ and $y = 9$. Therefore

$$\boxed{S = \{(9,8);(8,9)\}}$$

Exercise

Given the line $L: y = 2x + 4$ and the circle: $x^2 + y^2 - 4x - 6y - 12 = 0$.

1. Let us find the points of intersection of the line with the circle

$$\begin{cases} y = 2x + 4... \ (1) \\ x^2 + y^2 - 4x - 6y - 12 = 0... \ (2) \end{cases}$$

(1) into (2) yields $x^2+(2x+4)^2-4x-6(2x+4)-12=0$ the resolution gives $x=\pm 2...(3)$.

(3) into (1): for $x=2$, $y=8$ and for $x=-2$, we obtain $y=0$. The points of intersection are $A(-2,0)$ and $B(2,8)$.

2. Let us find the length define by theses two points:

$$\begin{aligned} AB &= \sqrt{(x_B-x_A)^2+(y_B-y_A)^2} \\ &= \sqrt{4^2+8^2} \\ &= 4\sqrt{5} \end{aligned}$$

Hence,

$$\boxed{AB = 4\sqrt{5}}$$

Exercise

Given the function $f(x)=(2x^2-7x+7)e^x$ let correct answer:

- $D_f=\mathbb{R}=]-\infty,+\infty[$
- For $x\to-\infty$, we have $(2x^2-7x+7)e^x\to 0$
- For $x\to+\infty$, we have $(2x^2-7x+7)e^x\to+\infty$
- $f(-\frac{1}{2})=(\frac{2}{4}+\frac{7}{2}+7)e^{-\frac{1}{2}}=\frac{11}{\sqrt{e}}$
- $f'(x)=(4x-7)e^x+(2x^2-7x+7)e^x=(2x^2-3x)e^x$
- $f'(x)=0$ is equivalent to $2x^2-3x=0$ since $e^x\neq 0$ this yields to $x=\frac{3}{2}$ and $x=0$
- $f(0)=7$
- $f(\frac{3}{2})=2e^{\frac{3}{2}}$
- The function f is positive and increasing in $]-\infty,\frac{1}{2}]$
- The minimum of the function f is 0.
- $f(x)\geq -e^2$ for all real number x.

  i.e we choose 3), 5) and 6) as the correct answers.