

REPUBLIQUE DU CAMEROUN
Paix- Travail- Patrie

REPUBLIC OF CAMEROON
Peace- Work- Fatherland

MINISTERE DES FORETS ET DE LA
FAUNE

MINISTRY OF FORESTRY AND
WILDLIFE

SECRETARIAT GENERAL

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CELESTIN DE L'ENSEIGNEMENT

EDUCATION UNIT

NATIONAL FORESTRY SCHOOL, MBALMAYO

COMPETITIVE ENTRANCE EXAMINATION

AUGUST 2005 Session 1

CYCLE B₂: Techniciens Supérieurs des Eaux et Forêts (TSEF)

Subject: MATHEMATICS

Time allowed: 4 hours

Coefficient: 5 or 3

Exercises:

I- (4mks)

A first box contains seven balls numbered from 1 to 7 and a second box contains four balls numbered 1 to 4. A ball is drawn at random from the first box and then from the second box.

1- What is the probability that the numbers drawn are identical? (1mk)

2- Let x be a random variable which denotes the absolute value of the difference between the two numbers

a- Determine the law of probability for x . (1,5mks)

b- Calculate the mathematical expectation and the variance of x (answers should be expressed as fractions or their simplest terms. (1,5mks)

✓ II- (5mks)

Consider the polynomial of the complex variable Z defined by:

$$P(Z) = 2Z^4 - 6Z^3 + 9Z^2 - 6Z + 2$$

1- Show that if Z_0 is a root of the polynomial, the numbers $\frac{1}{Z_0}$ and $\overline{Z_0}$ are also

roots of the polynomial. (2mks)

2- Calculate $(1+i)^2$; $(1+i)^3$; $(1+i)^4$ and $-(1+i)$ (1,5mks)

3- Hence solve the equation $P(Z) = 0$ in $\mathbb{C}$, which will have four distinct solutions. (1,5mks)

* PROBLEM (11mks)

Part A:

Let g be a numerical function of a real variable x defined on $]-2; +\infty[$ by :

$$g(x) = (x+2)^2 - 1 + \ln(x+2)$$

1- Calculate $g'(x)$ and hence deduce the direction of the variation of g (0,75mk)

2- Calculate $g(-1)$ and hence deduce the sign of $g(x)$, following the values of x . (0,75mk)
(the graphical representation of g is not necessary).

Deco

Part B :

Consider the numerical function f on a real variable x defined on $]-2, +\infty[$ by :

$$f(x) = x - \frac{\ln(x+2)}{x+2}$$

- 1- a) Determine the limits of $f(x)$ as x tends to -2^+ and when x tends to $+\infty$ (1mk)

b) Show that: $f'(x) = \frac{g(x)}{(x+2)^2}$ (0.5mk)

- c) Draw a variation table of f . (0.5mk)

2- Let (C) be the curve of f and (D) the equation of the straight line $y=x$.

- a- Show that (C) has a vertical asymptote and the line (D) as an oblique asymptote (1mk)

- b- Show that (C) and (D) meet at only one point and give the coordinates.

- c- Show that there exists a unique point (B) on the curve (C) where the tangent (T) is parallel to (D) . State the coordinates of B and the equation of (T) (1mk)

- d- Carefully draw the curve (C) specifying the asymptotes (vertical and oblique) and the tangent (T) on an orthonormal plane (O, i, j) . [unit of measurement of length on the axes: 2cm] (1mk)

[Take $e \approx 2,7$; $\frac{1}{e} \approx 0,37$; $\ln 2 \approx 0,7$ and $\ln 3 \approx 1$]

- 3- Let $\lambda \geq -1$; Calculate the area $A(\lambda)$, of the domain restricted by: the curve (C) , the line (D) and the straight line equations $x=-1$ and $x = \lambda$. Find the limit of $A(\lambda)$ as λ tends to $+\infty$? (1,5mk)

- 4- Let h be the restriction of f on the interval $[-1, +\infty[$.

- a- Justify that h is a bijection of $[-1, +\infty[$ on $[-1, +\infty[$. (0,5mk)

- b- Draw the curve (C') of h^{-1} , on the same graph as (C) . (Do not study the graph of h^{-1}). (1mk)

GOOD LUCK !!!!!